

AP[®] Exam Practice Questions for Chapter 7

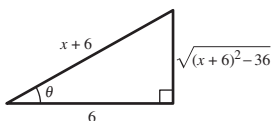
1. $u = 2 + \cos x \Rightarrow du = -\sin x \, dx$

$$\begin{aligned} \int (\sin x)e^{2+\cos x} \, dx &= -\int e^u \, du \\ &= -e^u + C \\ &= -e^{2+\cos x} + C \end{aligned}$$

So, the answer is A.

2. $\int \frac{1}{\sqrt{x^2 + 12x}} \, dx = \int \frac{1}{\sqrt{(x^2 + 12x + 36) - 36}} \, dx$

$$= \int \frac{1}{\sqrt{(x+6)^2 - 36}} \, dx$$



$$\sec \theta = \frac{x+6}{6} \Rightarrow x = 6 \sec \theta - 6, \, dx = 6 \sec \theta \tan \theta \, d\theta$$

$$\tan \theta = \frac{\sqrt{(x+6)^2 - 36}}{6} \Rightarrow \sqrt{(x+6)^2 - 36} = 6 \tan \theta$$

$$\begin{aligned} \int \frac{1}{\sqrt{x^2 + 12x}} \, dx &= \int \frac{1}{6 \tan \theta} \cdot 6 \sec \theta \tan \theta \, d\theta \\ &= \int \sec \theta \, d\theta \\ &= \ln|\sec \theta + \tan \theta| + C_1 \\ &= \ln \left| \frac{x+6}{6} + \frac{\sqrt{(x+6)^2 - 36}}{6} \right| + C_1 \\ &= \ln \left| \frac{x+6 + \sqrt{x^2 + 12x}}{6} \right| + C_1 \\ &= \ln|x+6 + \sqrt{x^2 + 12x}| - \ln 6 + C_1 \\ &= \ln|x+6 + \sqrt{x^2 + 12x}| + C_2 \end{aligned}$$

So, the answer is D.

3. $\lim_{x \rightarrow 1} \frac{5e^{1-x} - \ln x - 5}{x^2 - 1} = \lim_{x \rightarrow 1} \frac{-5e^{1-x} - (1/x)}{2x}$

$$\begin{aligned} &= \frac{-5e^0 - (1/1)}{2(1)} \\ &= \frac{-5 - 1}{2} \\ &= -3 \end{aligned}$$

So, the answer is A.

4. $u = f(x) \Rightarrow du = f'(x) \, dx$

$$dv = g'(x) \, dx \Rightarrow v = g(x)$$

$$\begin{aligned} \int_1^2 f(x)g'(x) \, dx &= [f(x)g(x)]_1^2 - \int_1^2 f'(x)g(x) \, dx \\ &= [f(2)g(2) - f(1)g(1)] - 3 \\ &= (2)(3) - (3)(-1) - 3 \\ &= 6 \end{aligned}$$

So, the answer is D.

$$5. \frac{1}{x^2 - 9x + 20} = \frac{1}{(x-5)(x-4)} = \frac{A}{x-5} + \frac{B}{x-4}$$

$$1 = A(x-4) + B(x-5)$$

When $x = 4$, $B = -1$.

When $x = 5$, $A = 1$.

$$\int_0^3 \frac{1}{x^2 - 9x + 20} dx = \int_0^3 \frac{1}{x-5} dx - \int_0^3 \frac{1}{x-4} dx$$

$$= [\ln|x-5| - \ln|x-4|]_0^3$$

$$= \left[\ln \left| \frac{x-5}{x-4} \right| \right]_0^3$$

$$= \ln 2 - \ln \frac{5}{4}$$

$$= \ln \frac{2}{\frac{5}{4}}$$

$$= \ln \frac{8}{5}$$

So, the answer is A.

$$8. \lim_{x \rightarrow \pi} \frac{\cos 2x - x \sin x - 1}{\pi^3 - x^3} = \lim_{x \rightarrow \pi} \frac{-2 \sin 2x - (x \cos x + \sin x)}{-3x^2}$$

$$= \lim_{x \rightarrow \pi} \frac{2 \sin 2x + x \cos x + \sin x}{3x^2}$$

$$= \frac{2 \sin 2\pi + \pi \cos \pi + \sin \pi}{3\pi^2}$$

$$= \frac{0 - \pi + 0}{3\pi^2}$$

$$= -\frac{1}{3\pi}$$

So, the answer is A.

9. Because $\lim_{x \rightarrow 0} \frac{f(x)}{g(x)} = \frac{0}{0}$, use L'Hôpital's Rule.

$$\lim_{x \rightarrow 0} \frac{f'(x)}{g'(x)} = \lim_{x \rightarrow 0} \frac{\sec^2 x}{4}$$

$$= \frac{\sec^2 0}{4}$$

$$= \frac{1}{4}$$

So, the answer is B.

$$6. \int_0^\pi \cos^3\left(\frac{x}{2}\right) dx = \int_0^\pi \cos^2\left(\frac{x}{2}\right) \cos\left(\frac{x}{2}\right) dx$$

$$= \int_0^\pi \left[1 - \sin^2\left(\frac{x}{2}\right)\right] \cos\left(\frac{x}{2}\right) dx$$

$$\text{Let } u = \sin\left(\frac{x}{2}\right), du = \frac{1}{2} \cos\left(\frac{x}{2}\right) dx$$

$$\int_0^\pi \cos^3\left(\frac{x}{2}\right) dx = 2 \int_0^1 (1 - u^2) du$$

$$= 2 \left[u - \frac{u^3}{3} \right]_0^1$$

$$= 2 \left(1 - \frac{1}{3} \right)$$

$$= \frac{4}{3}$$

So, the answer is A.

$$7. \int_0^\infty x e^{-x/3} dx = \lim_{b \rightarrow \infty} \int_0^b x e^{-x/3} dx$$

$$u = x \quad \Rightarrow \quad du = dx$$

$$dv = e^{-x/3} dx \quad \Rightarrow \quad v = -3e^{-x/3}$$

$$= \lim_{b \rightarrow \infty} \left([-3x e^{-x/3}]_0^b - \int_0^b -3e^{-x/3} dx \right)$$

$$= \lim_{b \rightarrow \infty} \left([-3x e^{-x/3}]_0^b + 3[-3e^{-x/3}]_0^b \right)$$

$$= \lim_{b \rightarrow \infty} \left((-3b e^{-b/3} - 0) - 9(e^{-b/3} - e^0) \right)$$

$$= 0 - 9(0 - 1)$$

$$= 9$$

So, the answer is C.

$$10. \int_1^\infty \frac{dx}{x^{5/2}} = \lim_{b \rightarrow \infty} \int_1^b x^{-5/2} dx = \lim_{b \rightarrow \infty} \left[-\frac{2}{3} x^{-3/2} \right]_1^b$$

$$= -\frac{2}{3} \lim_{b \rightarrow \infty} \left(\frac{1}{b^{3/2}} - 1 \right) = -\frac{2}{3}(0 - 1)$$

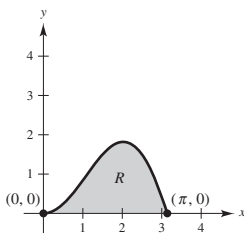
$$= \frac{2}{3}$$

So, the answer is C.

11. $V = 2\pi \int_2^3 x \left(\frac{\sqrt{36 - 4x^2}}{x} \right) dx = 2\pi \int_2^3 \sqrt{36 - 4x^2} dx \approx 19.46$

So, the answer is B.

12. (a)



$$A = \int_0^\pi x \sin x dx$$

$$u = x \Rightarrow du = dx$$

$$dv = \sin x dx \Rightarrow v = -\cos x$$

$$A = [-x \cos x]_0^\pi - \int_0^\pi -\cos x dx$$

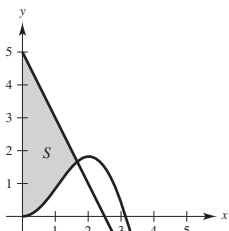
$$= [-x \cos x + \sin x]_0^\pi$$

$$= [-\pi(-1) + 0] - (0 - 0)$$

$$= \pi$$

So, the area is π .

(b)



Find the intersection of the graphs.

$$x \sin x = -2x + 5$$

$$x \approx 1.6693678$$

$$A = \int_0^{1.6693678} [(-2x + 5) - (x \sin x)] dx$$

$$= \int_0^{1.6693678} (-2x + 5) dx - \int_0^{1.6693678} x \sin x dx$$

$$u = x \Rightarrow du = dx$$

$$dv = \sin x dx \Rightarrow v = -\cos x$$

$$= [-x^2 + 5x]_0^{1.6693678} - [-x \cos x]_0^{1.6693678}$$

$$+ \int_0^{1.6693678} -\cos x dx$$

$$= [-x^2 + 5x + x \cos x - \sin x]_0^{1.6693678}$$

$$\approx 4.401$$

(c) $\int_0^{1.6693678} (-2x + 5 - x \sin x) dx$

$$= 2 \int_0^k (-2x + 5 - x \sin x) dx$$

3 pts: $\begin{cases} 2 \text{ pts: finds antiderivative (using parts)} \\ 1 \text{ pt: answer} \end{cases}$

Reminders: Be sure to justify the antiderivative by explicitly showing integration by parts—as per the given directions. (Merely writing down the definite integral and numerically approximating the area on your calculator will not receive full credit here because the directions say to use integration by parts.)

If presenting the answer as a decimal approximation, be sure to round the answer to at least three decimal places to receive credit on the exam.

3 pts: $\begin{cases} 2 \text{ pts: writes integral} \\ 1 \text{ pt: answer (no work needed)} \end{cases}$

Reminders: Use your calculator to find the point of intersection of the two graphs (no work needed for this).

In this intermediate step, round the coordinates of the intersection point to more than three decimal places to use in upcoming integrals.

Because the directions do not specify otherwise, it is sufficient here to simply write down the definite integral and then numerically approximate the area on your calculator.

Be sure to round the answer to at least three decimal places to receive credit on the exam.

3 pts: $\begin{cases} 1 \text{ pt: integrand} \\ 2 \text{ pts: limits of integration within correct equation} \end{cases}$

13. (a) $x(t) = 2 + \int_0^2 v(t) dt$
 $= 2 + \int_0^2 \left(te^{-t/3} - \frac{1}{2} \right) dt$
 $= 2 + \int_0^2 te^{-t/3} dt - \frac{1}{2} \int_0^2 dt$
 $u = t \quad \Rightarrow \quad du = dt$
 $dv = e^{-t/3} dt \quad \Rightarrow \quad v = -3e^{-t/3}$
 $= 2 + \left[-3te^{-t/3} \right]_0^2 + 3 \int_0^2 e^{-t/3} dt - \frac{1}{2} [t]_0^2$
 $= 2 + \left[-3te^{-t/3} - 9e^{-t/3} \right]_0^2 - 1$
 $= 1 + \left[(-3(2)e^{-2/3} - 9e^{-2/3}) - (0 - 9) \right]$
 $= 10 - \frac{15}{e^{2/3}}$
 ≈ 2.299

(b) $\int_0^{0.61344435} \left(te^{-t/3} - \frac{1}{2} \right) dt + \int_{0.61344435}^2 \left(te^{-t/3} - \frac{1}{2} \right) dt$

(c) $a(t) = e^{-t/3} + t \left(-\frac{1}{3} e^{-t/3} \right) - 0 = e^{-t/3} \left(1 - \frac{t}{3} \right)$

Because $a(3) = e^{-3/3} \left[1 - \frac{3}{3} \right] = 0$, the particle's speed is not changing at $t = 3$.

5 pts: $\begin{cases} 3 \text{ pts: finds antiderivative (using parts)} \\ 1 \text{ pt: uses initial condition} \\ 1 \text{ pt: answer} \end{cases}$

Reminders: Be sure to justify the antiderivative by explicitly showing integration by parts—as per the given directions. (Merely writing down the definite integral and numerically approximating the area on your calculator will not receive full credit here because the directions say to use integration by parts.)

Be sure to round your answer to at least three decimal places to receive credit on the exam.

2 pts: integrals

2 pts: $\begin{cases} 1 \text{ pt: computes } a(3) = v'(3) \text{ (no work)} \\ 1 \text{ pt: answer ("neither") with reason} \end{cases}$

14. (a) $f'(x) = \frac{1}{x(\ln x)^3}$

Because $f'(e) = \frac{1}{e(\ln e)^3} = \frac{1}{e}$, the equation

of the tangent line at $(e, 4)$ is

$$y - 4 = \frac{1}{e}(x - e)$$

$$y = \frac{1}{e}x + 3.$$

(b) $f'(x) = \frac{1}{x(\ln x)^3} = \frac{1}{x}(\ln x)^{-3}$

$$f''(x) = \frac{1}{x} \left[-3(\ln x)^{-4} \cdot \frac{1}{x} \right] + \left(-\frac{1}{x^2} \right) (\ln x)^{-3}$$

$$= -\frac{3}{x^2(\ln x)^4} - \frac{1}{x^2(\ln x)^3}$$

$$f''(e) = -\frac{3}{e^2(\ln e)^4} - \frac{1}{e^2(\ln e)^3} \approx -0.541 < 0$$

So, the graph of f is concave downward on $(1, 5)$

because $f''(x) < 0$ on $(1, 5)$.

(c) $f(x) = \int f'(x) dx = \int \frac{1}{x(\ln x)^3} dx$

$$u = \ln x \Rightarrow du = \frac{1}{x} dx$$

$$\int \frac{1}{x(\ln x)^3} dx = \int u^{-3} du$$

$$= -\frac{1}{2}u^{-2} + C$$

$$= -\frac{1}{2(\ln x)^2} + C$$

Use $(e, 4)$ to find C .

$$f(e) = -\frac{1}{2(\ln e)^2} + C = 4$$

$$-\frac{1}{2} + C = 4$$

$$C = \frac{9}{2}$$

$$\text{So, } f(x) = -\frac{1}{2(\ln x)^2} + \frac{9}{2}.$$

2 pts: $\begin{cases} 1 \text{ pt: finds the slope of the tangent line } [f'(e)] \\ 1 \text{ pt: writes an equation of the tangent line} \end{cases}$

4 pts: $\begin{cases} 2 \text{ pts: finds } f''(x) \\ 2 \text{ pts: answer (concave down) with reason [examines} \\ \text{sign of } f''(x) \text{ on this interval]} \end{cases}$

Notes: After finding $f''(x)$, you could simply state that $f''(x)$ is negative for all x -values on $(1, 5)$; a sign chart may not be necessary.

If using a sign chart as part of the justification, the functions $f''(x)$ and f must be explicitly labeled in your chart. Unlabeled sign charts may not receive credit on the exam.

A sign chart alone is generally *not* sufficient for the explanation. To receive full credit on the exam, explain the information contained in the sign chart. For example, state “ f is concave down on $(1, 5)$ because $f''(x)$ is negative on this interval.”

3 pts: $\begin{cases} 2 \text{ pts: antiderivative (shows substitution)} \\ 1 \text{ pt: uses } f(e) = 4 \text{ to find constant of} \\ \text{integration and particular solution} \end{cases}$

15. (a) Because $\lim_{x \rightarrow 1^+} (\sqrt{x} - 1) = 0$ and $\lim_{x \rightarrow 1^+} (x^2 - 1) = 0$,
L'Hôpital's Rule applies.

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} \frac{\sqrt{x} - 1}{x^2 - 1} = \lim_{x \rightarrow 1^+} \frac{\frac{1}{2\sqrt{x}}}{2x} = \frac{1}{4}$$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} \left[\tan^2(x - 1) + \frac{1}{4} \right] = \frac{1}{4}$$

$$\text{So, } \lim_{x \rightarrow 1} f(x) = \frac{1}{4}$$

$$f(1) = \tan^2(1 - 1) + \frac{1}{4} = \frac{1}{4}$$

Because $f(1) = \lim_{x \rightarrow 1} f(x)$, f is continuous at $x = 1$.

$$\begin{aligned} \text{(b) } f'(x) &= \begin{cases} \frac{(x^2 - 1) \left(\frac{1}{2\sqrt{x}} \right) - (\sqrt{x} - 1)(2x)}{(x^2 - 1)^2}, & x > 1 \\ 2 \tan(x - 1) \sec^2(x - 1), & x \leq 1 \end{cases} \\ &= \begin{cases} \frac{x^2 - 1 - 4x^2 + 4x\sqrt{x}}{2\sqrt{x}(x^2 - 1)^2}, & x > 1 \\ 2 \tan(x - 1) \sec^2(x - 1), & x \leq 1 \end{cases} \\ &= \begin{cases} \frac{-3x^2 + 4x\sqrt{x} - 1}{2\sqrt{x}(x^2 - 1)^2}, & x > 1 \\ 2 \tan(x - 1) \sec^2(x - 1), & x \leq 1 \end{cases} \end{aligned}$$

$$\begin{aligned} \text{(c) Average} &= \frac{1}{4} \int_0^4 f(x) dx \\ &= \frac{1}{4} \left[\int_0^1 \left(\tan^2(x - 1) + \frac{1}{4} \right) dx + \int_1^4 \frac{\sqrt{x} - 1}{x^2 - 1} dx \right] \\ &\approx 0.295 \end{aligned}$$

$$\begin{cases} 2 \text{ pts: finds } \lim_{x \rightarrow 1^+} f(x) \text{ using L'Hôpital's Rule} \\ 5 \text{ pts: } \begin{cases} 2 \text{ pts: finds } \lim_{x \rightarrow 1^-} f(x) \text{ and } \lim_{x \rightarrow 1} f(x) \\ 1 \text{ pt: answer with reason: shows } f(1) = \lim_{x \rightarrow 1} f(x) \end{cases} \end{cases}$$

Reminder: Be sure to first establish the indeterminate form $\frac{0}{0}$ to justify the use of L'Hôpital's Rule.

2 pts: derivatives of each piece

Reminder: You do not need to simplify these derivatives.

2 pts: sets up the definite integral for average value in two pieces

Note: You would not be expected to evaluate these definite integrals without a calculator.

$$16. (a) \frac{1}{2R(5-R)} = \frac{1}{2} \left[\frac{A}{R} + \frac{B}{5-R} \right]$$

$$A(5-R) + BR = 1 \Rightarrow A = \frac{1}{5} \text{ and } B = \frac{1}{5}$$

$$\int dT = \frac{1}{2} \int \frac{1}{R(5-R)} dR$$

$$T = \frac{1}{2} \int \left(\frac{1}{5R} + \frac{1}{5(5-R)} \right) dR$$

$$T = \frac{1}{10} (\ln|R| - \ln|5-R|) + C$$

$$T = \frac{1}{10} \ln \left| \frac{R}{5-R} \right| + C$$

Use $T = 0$ and $R = 3$ to find C .

$$0 = \frac{1}{10} \ln \left| \frac{3}{5-3} \right| + C$$

$$0 = \frac{1}{10} \ln \frac{3}{2} + C$$

$$C = -\frac{1}{10} \ln \frac{3}{2}$$

$$\text{So, } T = \frac{1}{10} \ln \left| \frac{R}{5-R} \right| - \frac{1}{10} \ln \frac{3}{2}.$$

$$(b) \frac{dT}{dR} = \frac{1}{2R(5-R)} = \frac{1}{10R - 2R^2}$$

$$\frac{d^2T}{dR^2} = \frac{(10R - 2R^2)(0) - 1(10 - 4R)}{(10R - 2R^2)^2}$$

$$= \frac{4R - 10}{(10R - 2R^2)^2}$$

$$0 = \frac{4R - 10}{(10R - 2R^2)^2}$$

$$R = \frac{5}{2}$$

$$\text{For } R = 2, \frac{d^2T}{dR^2} = \frac{4(2) - 10}{[10(2) - 2(2^2)]^2} = -\frac{1}{72}$$

$$\text{For } R = 3, \frac{d^2T}{dR^2} = \frac{4(3) - 10}{[10(3) - 2(3^2)]^2} = \frac{1}{72}$$

Because $\frac{d^2T}{dR^2}$ changes sign at $R = \frac{5}{2}$, the

graph of T has a point of inflection at $R = \frac{5}{2}$.

5 pts: $\left\{ \begin{array}{l} 3 \text{ pts: antiderivative (shows partial fraction decomposition)} \\ 1 \text{ pt: uses initial value} \\ 1 \text{ pt: answer} \end{array} \right.$

4 pts: $\left\{ \begin{array}{l} 1 \text{ pt: finds } \frac{d^2T}{dR^2} \\ 1 \text{ pt: finds where } \frac{d^2T}{dR^2} = 0 \\ 2 \text{ pts: answer with reason (examines sign of } \frac{d^2T}{dR^2} \text{ on both} \\ \text{sides of } R = \frac{5}{2} \text{ to confirm } T \text{ changes concavity)} \end{array} \right.$

Reminder: In such an explanation, reason using $\frac{d^2T}{dR^2}$

(explain using a derivative). Merely reasoning with T (appealing to where T changes concavity) may not receive credit on the exam.

Notes: If using a sign chart as part of the justification, the functions $\frac{d^2T}{dR^2}$ and T must be explicitly labeled in your chart. Unlabeled sign charts may not receive credit on the exam.

A sign chart alone is generally *not* sufficient for the explanation. To receive full credit on the exam, explain the information contained in the sign chart. For example, state “ T has an inflection point at $R = \frac{5}{2}$ because $\frac{d^2T}{dR^2}$

changes sign at $R = \frac{5}{2}$.”

17. (a) Because $\lim_{x \rightarrow \infty} \frac{x}{e^x} = \frac{\infty}{\infty}$, apply L'Hôpital's Rule.

$$\lim_{x \rightarrow +\infty} g(x) = \lim_{x \rightarrow +\infty} xe^{-x} = \lim_{x \rightarrow +\infty} \frac{x}{e^x} = \lim_{x \rightarrow +\infty} \frac{1}{e^x} = 0$$

Because $\lim_{x \rightarrow -\infty} \frac{x}{e^x} = \frac{-\infty}{0}$, L'Hôpital's Rule does not apply.

$$\lim_{x \rightarrow -\infty} g(x) = \lim_{x \rightarrow -\infty} xe^{-x} = (-\infty)(\infty) = -\infty$$

So, the graph of $g(x)$ has a horizontal asymptote at $y = 0$.

(b) $g(x) = xe^{-x}$

$$g'(x) = x(-e^{-x}) + e^{-x}(1)$$

$$= e^{-x}(-x + 1)$$

$$= \frac{1 - x}{e^x}$$

$$0 = \frac{1 - x}{e^x}$$

$$x = 1$$

$$\text{So, } g(1) = (1)e^{-(1)} = \frac{1}{e}.$$

$$\text{For } x = 0, g'(x) = \frac{1 - 0}{e^0} = 1$$

$$\text{For } x = 2, g'(x) = \frac{1 - 2}{e^2} = -\frac{1}{e^2}$$

Because $g'(x)$ changes sign at $x = 1$, the graph has a maximum at $\left(1, \frac{1}{e}\right)$.

(c) $A = \int_1^{\infty} xe^{-x} dx = \lim_{b \rightarrow \infty} \int_1^b xe^{-x} dx$

$$u = x \quad \Rightarrow \quad du = dx$$

$$dv = e^{-x} dx \quad \Rightarrow \quad v = -e^{-x}$$

$$A = \lim_{b \rightarrow \infty} \left([-xe^{-x}]_1^b - \int_1^b -e^{-x} dx \right)$$

$$= \lim_{b \rightarrow \infty} \left([-xe^{-x} - e^{-x}]_1^b \right)$$

$$= \lim_{b \rightarrow \infty} \left[(-be^{-b} - e^{-b}) - [-(1)e^{-1} - e^{-1}] \right]$$

$$= \lim_{b \rightarrow \infty} \left(-\frac{b}{e^b} - \frac{1}{e^b} + \frac{1}{e} + \frac{1}{e} \right)$$

$$= 0 - 0 + \frac{1}{e} + \frac{1}{e}$$

$$= \frac{2}{e}$$

3 pts: $\left\{ \begin{array}{l} 1 \text{ pt: considers both } \lim_{x \rightarrow \infty} g(x) \text{ and } \lim_{x \rightarrow -\infty} g(x) \\ 2 \text{ pts: applies L'Hôpital's Rule to evaluate the first limit } (x \rightarrow +\infty) \text{ but not the second limit } (x \rightarrow -\infty), \text{ answer} \end{array} \right.$

Reminder: Be sure to establish an indeterminate form, $\frac{0}{0}$ or $\frac{\infty}{\infty}$, to justify the use of L'Hôpital's Rule.

3 pts: $\left\{ \begin{array}{l} 1 \text{ pt: finds where } g'(x) = 0 \\ 2 \text{ pts: answer [finds } g(1)] \text{ with reason [examines sign of } g'(x) \text{ on both sides of } x = 1 \text{ to establish a relative maximum at the only critical value, } x = 1] \end{array} \right.$

Reminder: In such an explanation, reason using $g'(x)$ (explain using a derivative). Merely reasoning with g (appealing to where g changes from increasing to decreasing) may not receive credit on the exam.

Notes: If using a sign chart as part of the justification, the function $g'(x)$ and g must be explicitly labeled in your chart. Unlabeled sign charts may not receive credit on the exam.

A sign chart alone is generally *not* sufficient for the explanation. To receive full credit on the exam, explain the information contained in the sign chart. For example, state “ g has a relative maximum at $x = 1$ because $g'(x)$ changes from positive to negative at $x = 1$.”

3 pts: $\left\{ \begin{array}{l} 1 \text{ pt: sets up and rewrites improper integral} \\ 1 \text{ pt: antiderivative (shows parts)} \\ 1 \text{ pt: answer (evaluates limit using L'Hôpital's Rule)} \end{array} \right.$

Reminder: Be sure to establish an indeterminate form, $\frac{0}{0}$ or $\frac{\infty}{\infty}$, to justify the use of L'Hôpital's Rule.

$$\begin{aligned}
 18. (a) \int_0^3 (x-2)^{-2} dx &= \int_0^2 (x-2)^{-2} dx + \int_2^3 (x-2)^{-2} dx \\
 &= \lim_{b \rightarrow 2^-} \int_0^b (x-2)^{-2} dx + \lim_{b \rightarrow 2^+} \int_b^3 (x-2)^{-2} dx \\
 &= \lim_{b \rightarrow 2^-} \left[-(x-2)^{-1} \right]_0^b + \lim_{b \rightarrow 2^+} \left[-(x-2)^{-1} \right]_b^3 \\
 &= \lim_{b \rightarrow 2^-} \left(-\frac{1}{b-2} - \frac{1}{2} \right) + \lim_{b \rightarrow 2^+} \left(-1 + \frac{1}{b-2} \right) \\
 &= \infty + \infty \\
 &= \infty
 \end{aligned}$$

So, $\int_0^3 g'(x) dx$ diverges.

(b) The integrand has an infinite discontinuity at $x = 2$, which is contained in $[0, 3]$. So, the given integral is improper and the Fundamental Theorem of Calculus does not apply.

$$\begin{aligned}
 (c) \int_4^\infty (x-2)^{-2} dx &= \lim_{b \rightarrow \infty} \int_4^b (x-2)^{-2} dx \\
 &= \lim_{b \rightarrow \infty} \left[-(x-2)^{-1} \right]_4^b \\
 &= \lim_{b \rightarrow \infty} \left[-\frac{1}{b-2} + \frac{1}{2} \right] \\
 &= 0 + \frac{1}{2} \\
 &= \frac{1}{2}
 \end{aligned}$$

5 pts: $\left\{ \begin{array}{l} 2 \text{ pts: splits improper integral into two pieces} \\ \text{(accounts for discontinuity at } x = 2) \\ \text{with correct one-sided limits} \\ 1 \text{ pt: antiderivative} \\ 2 \text{ pts: shows each one-sided limit goes to} \\ \text{infinity, answer} \end{array} \right.$

1 pt: reason [identifies $g'(x)$ has a discontinuity in $[0, 3]$ at $x = 2$]

3 pts: $\left\{ \begin{array}{l} 1 \text{ pt: rewrites improper integral with limit} \\ 1 \text{ pt: antiderivative} \\ 1 \text{ pt: answer (evaluates limit)} \end{array} \right.$

19. (a) Because $\lim_{x \rightarrow 0} \frac{f(x) + 2}{\tan x} = \frac{0}{0}$, apply L'Hôpital's Rule.

$$\lim_{x \rightarrow 0} \frac{f'(x)}{\sec^2 x} = \frac{f'(0)}{1} = (-2)^2[4(0) + 1] = 4$$

(b) $f\left(\frac{1}{4}\right) \approx -2 + \frac{1}{4}(4) = -1$

$$f\left(\frac{1}{2}\right) \approx -1 + \frac{1}{4}(2) = -\frac{1}{2}$$

So, $f\left(\frac{1}{2}\right) \approx -\frac{1}{2}$.

(c) $\frac{dy}{dx} = y^2(4x + 1)$

$$\frac{1}{y^2} dy = (4x + 1) dx$$

$$-\frac{1}{y} = 2x^2 + x + C$$

Use $f(0) = -2$ to find C .

$$-\frac{1}{(-2)} = 2(0)^2 + 0 + C \Rightarrow C = \frac{1}{2}$$

Because $-\frac{1}{y} = 2x^2 + x + \frac{1}{2}$,

$$f(x) = -\left(2x^2 + x + \frac{1}{2}\right)^{-1}.$$

2 pts: $\left\{ \begin{array}{l} 1 \text{ pt: applies L'Hôpital's Rule} \\ 1 \text{ pt: answer} \end{array} \right.$

Reminder: Be sure to establish an indeterminate form, $\frac{0}{0}$ or $\frac{\infty}{\infty}$, to justify the use of L'Hôpital's Rule.

2 pts: $\left\{ \begin{array}{l} 1 \text{ pt: Euler's Method with two steps} \\ 1 \text{ pt: answer [approximation of } f(0.5)] \end{array} \right.$

Reminder: Be sure to write " $f(0.5) \approx$ " rather than " $f(0.5) =$." Because this is an approximation, a point may be deducted if an equal sign is used. In general, equating two quantities that are not truly equal will result in a one point deduction on a free-response question.

5 pts: $\left\{ \begin{array}{l} 1 \text{ pt: separation of variables} \\ 1 \text{ pt: antiderivatives} \\ 1 \text{ pt: constant of integration} \\ 1 \text{ pt: uses initial condition} \\ 1 \text{ pt: particular solution (solves for } y) \end{array} \right.$

Notes: 2 points max if no constant of integration present

0 points if no separation of variables