AP® Exam Practice Questions for Chapter 4

1. Use the Trapezoidal Rule to find an approximation of $\int_0^4 g(x)\,dx$.

$$\int_0^4 g(x)\,dx \approx \frac{4-0}{2(8)}\left[0 + 2(3) + 2(7) + 2(12) + 2(18) + 2(25) + 2(33) + 2(42) + 52\right]$$
$$= \frac{1}{4}(332) = 83$$

So, the answer is B.

2. $$\int_0^5 f(x)\,dx = \int_0^3 f(x)\,dx + \int_3^5 f(x)\,dx$$
$$= \frac{1}{2}(3+5)3 + 2(-2)$$
$$= 8$$

So, the answer is C.

3. $$\int \frac{4}{(x-5)^2 + 9}\,dx$$

Let $u = x - 5$, $du = dx$, and $a = 3$.

$$4\int \frac{1}{u^2 + a^2}\,dx = 4\left[\frac{1}{a}\arctan\frac{u}{a} + C\right]$$
$$= 4\left(\frac{1}{3}\arctan\frac{x-5}{3} + C\right)$$
$$= \frac{4}{3}\tan^{-1}\frac{x-5}{3} + C$$

So, the answer is A.

4. $v(t) = 4t^3 - 4t$, $0 \le t \le 2$

$$\frac{1}{2-0}\int_0^2 v(t)\,dt = \frac{1}{2}\int_0^2 \left(4t^3 - 4t\right)dt$$
$$= \frac{1}{2}\left[t^4 - 2t^2\right]_0^2$$
$$= \frac{1}{2}(16 - 8)$$
$$= 4 \text{ units/sec}$$

So, the answer is A.

5. $$g(-1) = \int_4^{-1} f(t)\,dt$$
$$= -\int_{-1}^4 f(t)\,dt$$
$$= -\int_{-1}^0 f(t)\,dt - \int_0^3 f(t)\,dt - \int_3^4 f(t)\,dt$$
$$= -\tfrac{1}{2}(-2) - \tfrac{1}{2}(4)(-2) - \tfrac{1}{2}(2)$$
$$= 4$$

So, the answer is C.

6. $$\int_b^\pi \sin x\,dx = 0.4$$
$$\left[-\cos x\right]_b^\pi = 0.4$$
$$-\cos\pi + \cos b = 0.4$$
$$1 + \cos b = 0.4$$
$$\cos b = -0.6$$
$$b \approx 2.214$$

So, the answer is D.

7. $f'(x) = \sqrt{x^3 + 6}$

$a = 1$, $b = 5$

$$f(b) = f(a) + \int_a^b f'(x)\,dx$$
$$f(5) = f(1) + \int_1^5 \sqrt{x^3 + 6}\,dx$$
$$\approx 2 + 24.672$$
$$= 26.672$$

So, the answer is D.

8. $$\text{Average value} = \frac{1}{\frac{3\pi}{2} - 0}\int_0^{3\pi/2}\left(\frac{2x}{x^2+1} + \sin x\right)dx = \frac{2}{3\pi}\int_0^{3\pi/2}\left(\frac{2x}{x^2+1} + \sin x\right)dx$$
$$= \frac{2}{3\pi}\left[\ln\left|x^2 + 1\right| - \cos x\right]_0^{3\pi/2} = \frac{2}{3\pi}\left[\ln\left|\frac{9\pi^2}{4} + 1\right| - (-1)\right]$$
$$\approx 0.879$$

So, the answer is B.

9. (a) $\int_0^{12} C'(t)\,dt = C(12) - C(0)$

$= 40 - 65$

$= -25°\text{C}$

The total temperature lost from $t = 0$ to $t = 12$ is 25°C.

2 pts: {1 pt: answer; 1 pt: units and interpretation of answer}

Note: Could simply explain as "change in temperature from time $t = 0$ to $t = 12$."

(b) $\frac{1}{12}\int_0^{12} C(t)$ represents the average temperature for $0 \le t \le 12$.

$$\frac{1}{12}\int_0^{12} C(t)\,dt = \frac{1}{12}\left[\int_0^3 C(t)\,dt + \int_3^5 C(t)\,dt + \int_5^7 C(t)\,dt + \int_7^8 C(t)\,dt + \int_8^{12} C(t)\,dt\right]$$

$$= \frac{1}{12}\left[\frac{3-0}{2}(65+57) + \frac{5-3}{2}(57+50) + \frac{7-5}{2}(50+46) + \frac{8-7}{2}(46+44) + \frac{12-8}{2}(44+40)\right]$$

$$\approx 49.917°\text{C}$$

So, the average temperature of the coffee is about 49.917°C.

3 pts: {1 pt: identifies as average temperature; 2 pts: answer with units and analysis}

(c) $C'(4) \approx \frac{C(5) - C(3)}{5 - 3} = \frac{50 - 57}{2}$

$= -3.5$

So, when $t = 4$, the temperature of the coffee is changing about $-3.5°\text{C}$ per minute.

1 pt: difference quotient with units

(d) $C(t) = \int C'(t)\,dt = \int -2\cos 0.5t\,dt$

$= -2\left(\frac{1}{0.5}\right)\int \cos(0.5t)(0.5)\,dt$

$= -4(\sin 0.5t) + K$

$= -4\sin 0.5t + K$

Because $C(t)$ is continuous at $C = 12$, use $C(12) = 40$ to find K.

$-4\sin[0.5(12)] + K = 40$

$K \approx 38.8823$

So, $C(15) = -4\sin(0.5 \cdot 15) + 38.8823$

$\approx 35.130°\text{C}.$

3 pts: {1 pt: antiderivative; 1 pt: uses $C(12) = 40$ to find constant of integration; 1 pt: finds $C(15)$ with units}

Reminder: Use more than three decimal places when representing the constant of integration (avoid premature rounding in this intermediate step) so that the final answer can be rounded to three decimal places. Perhaps store the value of the constant in your calculator for use in the subsequent computation.

Reminder: Round each answer to at least three decimal places to receive credit on the exam.

10. (a) $\int_0^6 M(t)\,dt = \int_0^6 \frac{\pi}{6} \sin \frac{\pi t}{12}\,dt$

$$= 2\int_0^6 \sin \frac{\pi - 6}{12}\left(\frac{\pi}{12}\right)dt$$

$$= 2\left[-\cos \frac{\pi t}{12}\right]_0^6$$

$$= 2\left[0 - (-1)\right]$$

$$= 2$$

So, 2 inches of snow will melt during the 6 hour period.

2 pts: { 1 pt: definite integral $\left[\text{writing } \int_0^6 M(t)\,dt \text{ is sufficient}\right]$; 1 pt: answer with units }

(b) $S(3) - M(3)$

$$= \left[0.006(9) - 0.12(3) + 0.87\right] - \left[\frac{\pi}{6} \sin \frac{\pi}{4}\right]$$

$$\approx 0.194 \text{ in./h}$$

2 pts: { 1 pt: considers $S(3) - M(3)$; 1 pt: answer with units }

(c) $I(t) = \int \left[s(t) - M(t)\right] dt$

$$= \int \left[\left(0.006t^2 - 0.12t + 0.87\right) - \frac{\pi}{6} \sin \frac{\pi t}{12}\right] dt$$

$$= 0.002t^3 - 0.06t^2 + 0.87t + 2\cos \frac{\pi t}{12} + C$$

$I(0) = 40 = 2 + C \Rightarrow C = 38$

$I(t) = 0.002t^3 - 0.06t^2 + 0.87t + 2\cos \frac{\pi t}{12} + 38$

3 pts: { 1 pt: represents $I(t)$ as an indefinite integral; 1 pt: uses initial condition to find constant of integration; 1 pt: answer }

(d) $I'(t) = 0 \Rightarrow S(t) = M(t)$

$t \approx 4.2406$ hours

(using a graphing utility)

$I(4.2406) \approx 41.6519$ inches

2 pts: { 1 pt: finds where $I'(t) = 0$ [i.e., where $S(t) = M(t)$]; 1 pt: answers with units }

Reminders:

Round each answer to at least three decimal places to receive credit.

No work is needed for these computations (after setting each up); use your calculator to compute/evaluate/solve.

11. (a) $v(t) = \sin \frac{\pi t}{4}$

The particle is moving to the right on the t-intervals $(0, 4)$ and $(8, 9)$ because $v(t) > 0$ on these intervals.

2 pts: answers with reason $\left[\text{identifies where } v(t) > 0\right]$

Reminder: Be sure to explicitly identify each function by name. Referring to $v(t)$ as "it," "the function," or "the graph" may not receive credit on the exam.

(b) $\int_0^4 \sin \frac{\pi t}{4}\,dt - \int_4^8 \sin \frac{\pi t}{4}\,dt + \int_8^9 \sin \frac{\pi t}{4}\,dt$

1 pt: definite integrals

(c) $v'(t) = \frac{\pi}{4} \cos \frac{\pi t}{4}$

$$a(3) = v'(3) = \frac{\pi}{4} \cos \frac{\pi(3)}{4} \approx -0.555$$

Because $a(3) < 0$ and $v(3) > 0$, the acceleration and velocity are in opposite directions. This means that the particle is slowing down.

3 pts: $\begin{cases} \text{1 pt: computes } a(t) = v'(t) \text{ (Chain Rule)} \\ \text{1 pt: answer for } a(3) \\ \text{1 pt: answer ("slowing down") with explanation} \end{cases}$

Reminder: $a(3)$ does *not* need to be evaluated or simplified. Leaving the answer as $\frac{\pi}{4} \cos \frac{3\pi}{4}$ or $\frac{\pi}{4}\left(-\frac{\sqrt{2}}{2}\right)$ is sufficient.

(d)

$$s(t) = \int v(t)\,dt = \int \sin \frac{\pi t}{4}\,dt = \frac{4}{\pi} \int \sin \frac{\pi t}{4}\left(\frac{\pi}{4}\right) dt = -\frac{4}{\pi} \cos \frac{\pi t}{4} + C$$

Use $s(0) = -4$ to find C.

$$-4 = -\frac{4}{\pi} \cos 0 + C$$

$$-4 + \frac{4}{\pi} = C$$

So, $s(t) = -\frac{4}{\pi} \cos \frac{\pi t}{4} + \frac{4 - 4\pi}{\pi}$.

Therefore, $s(3) = -\frac{4}{\pi} \cos \frac{3\pi}{4} + \frac{4 - 4\pi}{\pi} \approx -1.83$.

3 pts: $\begin{cases} \text{1 pt: antiderivative} \\ \text{1 pt: uses initial condition to find constant of integration} \\ \text{1 pt: answer} \end{cases}$

Reminder: The answer does not need to be evaluated or simplified.

12. $F(x) = \int_3^x f(t)\,dt$

(a) $F(0) = \int_3^0 f(t)\,dt$

$= -\int_0^3 f(t)\,dt$

$= -\left[\frac{1}{4}\pi(2)^2 + \frac{1}{2}(2 + 3)(1)\right]$

$= -\pi - \dfrac{5}{2}$

$F'(0) = f(0) = 3$

$F(4) = \int_3^4 f(t)\,dt$

$= \frac{1}{2}(-1)(1) = -0.5$

3 pts:
- 1 pt: answer for $F(0)$ with justification (computes signed area)
- 1 pt: answer for $F'(0)$ with justification [indicates $F'(0) = f(0)$]
- 1 pt: answer for $F(4)$ with justification (computes signed area)

(b) The graph of $F(x)$ does not have a minimum value because $F'(x) = f(x)$ does not change from negative to positive at any point.

2 pts: answer with reason [identifies that $F'(x) = f(x)$ does not change from negative to positive on this interval]

(c) Because $F'(x) = f(x)$ changes from increasing to decreasing at $x = 0$, an inflection point of the graph of $F(x)$ is $x = 0$.

2 pts: answer with reason [identifies where $F'(x) = f(x)$ changes from increasing to decreasing]

(d) Because $F'(2) = f(2) = 1$, the slope of the tangent line is 1. Use $F(2) = \int_3^2 f(t)\,dt = -\frac{1}{2}$ and $m = 1$ to write the equation of the tangent line.

$y + \frac{1}{2} = 1(x - 2)$

$y = x - \frac{5}{2}$

So, the equation of the tangent line at $F(2)$ is $y = x - \frac{5}{2}$.

2 pts:
- 1 pt: finds *slope* of the line tangent to $F(x)$ at $x = 2$ [finds $F'(2) = f(2)$]
- 1 pt: finds $F(2)$ (signed area) and an *equation* of the line tangent to F at $x = 2$

13. (a)

$$\int_0^1 f(x)\,dx = -5.5$$
$$F(1) - F(0) = -5.5$$
$$9 - F(0) = -5.5$$
$$F(0) = 14.5$$

$$\int_1^3 f(x)\,dx = -6$$
$$F(3) - F(1) = -6$$
$$F(3) - 9 = -6$$
$$F(3) = 3$$

$$\int_3^4 f(x)\,dx = 15.5$$
$$F(4) - F(3) = 15.5$$
$$F(4) - 3 = 15.5$$
$$F(4) = 18.5$$

So, $F(0) = 14.5$ and $F(4) = 18.5$.

4 pts:
- 2 pts: finds $F(0)$ with justification [uses $\int_0^1 f(x)\,dx = -5.5$ and $F(1) = 9$]
- 2 pts: finds $F(4)$ with justification [uses $\int_1^3 f(x)\,dx = -6$ and $F(1) = 9$ to find $F(3)$, and then uses $\int_3^4 f(x)\,dx = 15.5$ and $F(3) = 3$ to find $F(4)$]

(b) Because $F(0) > 5 > F(3)$ and $F(x)$ is continuous on $[0, 3]$, there is at least one x-value in this interval where $F(x) = 5$ by the Intermediate Value Theorem. Because $F(3) < 5 < F(4)$ and $F(x)$ is continuous on $[3, 4]$, there is at least one x-value in this interval where $F(x) = 5$. So, $F(x)$ must equal 5 at least two times on $[0, 4]$.

3 pts: answer with reason [appeals to values of $F(0)$, $F(3)$, and $F(4)$ and the continuity of $F(x)$]

(c) Because $F'(x) = f(x) > 0$ on the interval $(3, 4)$, F is increasing on the interval $(3, 4)$.

2 pts: answer with reason [appeals to where $F'(x) = f(x) > 0$]