AP® Exam Practice Questions for Chapter 2

1. $f(x) = 4e^x - x + 6$

$f'(x) = 4e^x - 1 + 0 = 4e^x - 1$

$f'(0) = 4e^0 - 1 = 4(1) - 1 = 3$

Tangent line: $y - 10 = 3(x - 0)$

$y = 3x + 10$

So, the answer is D.

2. Evaluate each graph.

A: The graph appears to be $f(x) = -2x$, so $f'(x) = -2$.

B: The graph appears to be $f(x) = x^3$, so $f'(x) = 3x^2$.

C: The graph appears to be $f(x) = x^2$, so $f'(x) = 2x$.

D: The graph appears to be $f(x) = x$, so $f'(x) = 1$.

Because $f'(x) = -2$ is negative for all values of x, the answer is A.

3. $y = \dfrac{6x^4 - 3x^5 + 5x^3}{x^3}$

$= 6x - 3x^2 + 5$

$\dfrac{dy}{dx} = 6 - 6x$

$\dfrac{d^2y}{dx^2} = -6$

So, the answer is D.

4. The function $h(x) = |2x - 5|$ is continuous at $x = \frac{5}{2}$. Because $\lim_{x\to(5/2)^-} h(x) \neq \lim_{x\to(5/2)^+} h(x)$, the function is not differentiable.

So, the answer is A.

5. $f(x) = \dfrac{\sin x}{x^2}$

$f'(x) = \dfrac{x^2(\cos x) - \sin x(2x)}{(x^2)^2} = \dfrac{x\cos x - 2\sin x}{x^3}$

So, the answer is C.

6. $y = \sqrt[4]{8x + 3} = (8x + 3)^{1/4}$

$y' = \dfrac{1}{4}(8x + 3)^{-3/4}(8) = \dfrac{2}{(8x + 3)^{3/4}}$

So, the answer is A.

7. $y = 6\cos 2x$

$y' = 6(-\sin 2x)(2) = -12\sin 2x$

$y'' = -12\cos 2x(2) = -24\cos 2x$

$y''' = -24(-\sin 2x)(2) = 48\sin 2x$

$y^{(4)} = 48\cos 2x(2) = 96\cos 2x$

$y^{(5)} = 96(-\sin 2x)(2) = -192\sin 2x$

$y^{(6)} = -192\cos 2x(2) = -384\cos 2x$

So, the answer is B.

8. $\dfrac{s(3.2) - s(2.7)}{3.2 - 2.7} = \dfrac{10.6 - 7.8}{3.2 - 2.7} = \dfrac{2.8}{0.5} = 5.6 \text{ m/sec}$

So, the answer is C.

9. $2y^3 - 3xy + x^2 = 4$

$6y^2\dfrac{dy}{dx} - 3x\dfrac{dy}{dx} - 3y + 2x = 0$

$(-3x + 6y^2)\dfrac{dy}{dx} = -2x + 3y$

$\dfrac{dy}{dx} = \dfrac{-(2x - 3y)}{(-3x + 6y^2)}$

$\dfrac{dy}{dx} = \dfrac{2x - 3y}{3x - 6y^2}$

So, the answer is B.

10. $V = \pi r^2 h$

$\dfrac{dV}{dt} = \pi\left[r^2\left(\dfrac{dh}{dt}\right) + 2r\left(\dfrac{dr}{dt}\right)h\right]$

$= \pi\left(r^2 \cdot \dfrac{dh}{dt} + 2rh\dfrac{dr}{dt}\right)$

When $\dfrac{dr}{dt} = \dfrac{1}{3}$, $\dfrac{dh}{dt} = \dfrac{1}{2}$, $h = 9$, and $r = 4$,

$\dfrac{dV}{dt} = \pi\left[(4)^2\left(\dfrac{1}{2}\right) + 2(4)(9)\left(\dfrac{1}{3}\right)\right]$

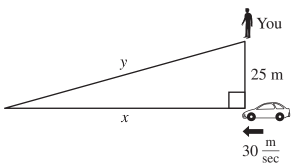
$= 32\pi$ cubic centimeters per second.

So, the answer is D.

11. $$\lim_{h\to 0}\frac{\sqrt{16+h}-4}{h} = \lim_{h\to 0}\frac{\sqrt{16+h}-4}{h}\cdot\frac{\sqrt{16+h}+4}{\sqrt{16+h}+4}$$
$$= \lim_{h\to 0}\frac{16+h-16}{h\left(\sqrt{16+h}+4\right)}$$
$$= \lim_{h\to 0}\frac{1}{h\left(\sqrt{16+h}+4\right)}$$
$$= \lim_{h\to 0}\frac{1}{\sqrt{16+h}+4}$$
$$= \frac{1}{\sqrt{16}+4}$$
$$= \frac{1}{8}$$

So, the answer is C.

12.



$$x^2 + 25^2 = y^2$$
$$2x\frac{dx}{dt} + 0 = 2y\frac{dy}{dt}$$
$$\frac{dy}{dt} = \frac{x}{y}\frac{dx}{dt}$$

After 3 seconds, $x = 30(3) = 90$ meters and $y = \sqrt{90^2 + 25^2} = \sqrt{8725}$.

Because $\frac{dx}{dt} = 30$ meters per second, $\frac{dy}{dt} = \frac{90}{\sqrt{8725}}(30) \approx 28.906$ meters per second.

So, the answer is B.

13. $s(t) = -t^3 + 2t^2 + \frac{3}{2}$

$s'(t) = -3t^2 + 4t$

Average velocity $= \frac{s(4) - s(0)}{4 - 0} = \frac{\left[-(4)^3 + 2(4)^2 + \frac{3}{2}\right] - \left[-(0)^3 + 2(0)^2 + \frac{3}{2}\right]}{4} = \frac{-32}{4} = -8$

$$s'(t) = -8$$
$$-3t^2 + 4t = -8$$
$$0 = 3t^2 - 4t - 8$$
$$t = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-8)}}{2(3)}$$
$$t = \frac{4 \pm \sqrt{112}}{6}$$
$$t \approx 2.431 \text{ seconds} \quad (t \approx -1.097 \text{ is not in the domain.})$$

So, the answer is D.

14. $v(t) = 2 + 3.5\cos(0.7t)$

(a) $v'(t) = -2.45\sin(0.7t)$

$v'(t)$ represents the particle's acceleration at time t.

3 pts: $\begin{cases} \text{2 pts: computes } v'(t) \\ \text{1 pt: answer (acceleration at time } t\text{)} \end{cases}$

(b) $f(t)$ represents the particle's position at time t.

1 pt: answer (position at time t)

(c) $a(t) = v'(t) = -2.45\sin(0.7t)$

$a(5) = -2.45\sin(3.5) \approx 0.859$

1 pt: answer [computes $a(5)$]

(d)

$$0 = 2 + 3.5\cos(0.7t)$$
$$-2 = 3.5\cos(0.7t)$$
$$-\frac{2}{3.5} = \cos(0.7t)$$
$$\cos^{-1}\left(-\frac{4}{7}\right) = 0.7t$$
$$\frac{10}{7}\cos^{-1}\left(-\frac{4}{7}\right) = t$$

Yes, the particle's velocity is 0 when $t = \frac{10}{7}\cos^{-1}\left(-\frac{4}{7}\right)$.

4 pts: $\begin{cases} \text{1 pt: sets } v(t) = 0 \\ \text{2 pts: shows a solution exists [solves for } t \text{ or examines signs of } v(t)] \\ \text{1 pt: answer} \end{cases}$

15. (a)

$$h(x) = \frac{f(x)}{g(x)}$$

$$h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{\left[g(x)\right]^2}$$

$$h'(2) = \frac{g(2)f'(2) - f(2)g'(2)}{\left[g(2)\right]^2}$$

$$= \frac{(5)(1) - (-3)(-2)}{5^2}$$

$$= -\frac{1}{25}$$

3 pts: $\begin{cases} \text{2 pts: uses the Quotient Rule to compute } h'(x) \\ \text{1 pt: answer} \end{cases}$

Note: Leaving the answer as $\frac{(5)(1) - (-3)(-2)}{5^2}$ is perfectly acceptable and, in fact, recommended. It is not necessary to simplify the answer on a free-response question. Students are encouraged to not simplify answers on the free-response questions to avoid the risk of making arithmetic mistakes.

(b)

$$j(x) = f(g(x))$$

$$j'(x) = f'(g(x))g'(x)$$

$$j'(2) = f'(g(2))g'(2)$$

$$= f'(5)(-2)$$

$$= (7)(-2)$$

$$= -14$$

3 pts: $\begin{cases} \text{2 pts: uses the Chain Rule to compute } j'(x) \\ \text{1 pt: answer} \end{cases}$

Note: Leaving the answer as $(7)(-2)$ is perfectly acceptable and, in fact, recommended. It is not necessary to simplify the answer on a free-response question.

(c)

$$k(x) = \sqrt{f(x)}$$

$$k'(x) = \frac{1}{2\sqrt{f(x)}} \cdot f'(x)$$

$$k'(5) = \frac{1}{2\sqrt{f(5)}} \cdot f'(5)$$

$$= \frac{1}{2\sqrt{4}} \cdot 7$$

$$= \frac{7}{4}$$

3 pts: $\begin{cases} \text{2 pts: uses the Chain Rule to compute } k'(x) \\ \text{1 pt: answer} \end{cases}$

Note: Leaving the answer as $\frac{1}{2\sqrt{4}} \cdot 7$ is perfectly acceptable and, in fact, recommended. It is not necessary to simplify the answer on a free-response question.

16. (a) (i) Because $v(t) > 0$ when $0 < t < 1$ and $4.4 < t < 5$, the particle is moving upward on the intervals $(0, 1)$ and $(4.4, 5)$.

(ii) Because $v(t) < 0$ when $2 < t < 4.4$, the particle is moving downward on the interval $(2, 4.4)$.

(iii) Because $v(t) = 0$ when $1 < t < 2$, the particle is at rest on the interval $(1, 2)$.

4 pts:
- 2 pts: (i) answers with explanation, where $v(t) > 0$
- 1 pt: (ii) answer with explanation, where $v(t) < 0$
- 1 pt: (iii) answer with explanation, where $v(t) = 0$

Notes: Though open intervals are desired, open or closed intervals would generally be accepted here.

For (i) and (ii): Because the zero of $v(t)$ on the interval $[4, 5]$ is being determined visually from the given graph, any t-value in the interval $[4.3, 4.5]$ would be acceptable for this zero. (Using exactly $t = 4.4$ in the answers is not required.)

(b) (i) When $t = 0.75$, the slope of the line of $v(t) = -2$. So, the acceleration of the particle is -2 feet per second squared.

(ii) When $t = 4.2$, the slope of the line of $v(t) = 5$. So, the acceleration of the particle is 5 feet per second squared.

5 pts:
- 2 pts: (i) answer with analysis (computes slope of this line segment)
- 2 pts: (ii) answer with analysis (computes slope of this line segment)
- 1 pt: units for both (i) and (ii)

Note: Leaving the answers in unsimplified forms, such as $\frac{2 - 0}{0 - 1}$ and $\frac{3 - (-2)}{5 - 4}$, is perfectly acceptable and, in fact, recommended. It is not necessary to simplify the answers on a free-response question.

17. (a) $g(x) = f(x) \cdot \tan x + kx$

$g'(x) = f'(x) \cdot \tan x + \sec^2 x \cdot f(x) + k$

Because $\tan \frac{\pi}{2}$ and $\tan \frac{3\pi}{2}$ are undefined and $\sec^2 \frac{\pi}{2}$ and $\sec^2 \frac{3\pi}{2}$ are undefined, the derivative of g will fail to exist when $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$.

7 pts:
- 3 pts: uses the Product Rule to compute $g'(x)$
- 2 pts: answers
- 2 pts: justification (explaning $\tan x$ and/or $\sec x$ are undefined here)

(b) $g'(x) = f'(x) \cdot \tan x + \sec^2 x \cdot f(x) + k$

$$g'\left(\frac{\pi}{4}\right) = f'\left(\frac{\pi}{4}\right) \cdot \tan \frac{\pi}{4} + \left(\sec \frac{\pi}{4}\right)^2 \cdot f\left(\frac{\pi}{4}\right) + k$$

$$6 = (-2)(1) + \left(\sqrt{2}\right)^2(4) + k$$

$$6 = 6 + k$$

$$k = 0$$

2 pts:
- 1 pt: sets up correct equation
- 1 pt: answer (solves for k)

Note: Leaving the answer as $k = 6 - (-2) \tan \frac{\pi}{4} - \left(\sec \frac{\pi}{4}\right)^2(4)$ is perfectly acceptable and, in fact, recommended. It is not necessary to simplify the answer on a free-response question.