AP® Exam Practice Questions for Chapter 1

1. $\lim_{x\to\pi} h(x) = \lim_{x\to\pi} 2 = 2$

The answer is B.

2. $\lim_{x\to-4^-} g(x) = \lim_{x\to-4^-} \frac{|x+4|}{x+4} = -1$

$\lim_{x\to-4^+} g(x) = \lim_{x\to-4^+} \frac{|x+4|}{x+4} = 1$

Because $\lim_{x\to-4^-} g(x) \neq \lim_{x\to-4^+}$, the limit is nonexistent.

The answer is D.

3. $\lim_{x\to\pi} \frac{\sin x}{x} = \frac{\sin \pi}{\pi} = \frac{0}{\pi} = 0$

The answer is A.

4. $\lim_{x\to-2} \frac{3x^2 + 5x + 7}{x - 4}$

$= \frac{3(-2)^2 + 5(-2) + 7}{(-2) - 4}$

$= \frac{9}{-6}$

$= -\frac{3}{2}$

The answer is B.

5. $\lim_{x\to5} [5f(x) - g(x)] = \lim_{x\to5} 5f(x) - \lim_{x\to5} g(x)$

$= 5\lim_{x\to5} f(x) - \lim_{x\to5} g(x)$

$= 5(10) - (1) = 49$

The answer is D.

6. $\lim_{x\to\infty} \frac{5x + 16x^2}{4x^2 - 3} = \lim_{x\to\infty} \frac{\frac{5}{x} + 16}{4 - \frac{3}{x^2}}$

$= \frac{0 + 16}{4 - 0}$

$= 4$

The answer is C.

7. $\lim_{x\to\infty} \frac{3}{1 - 4^x} = 0$

$\lim_{x\to-\infty} \frac{3}{1 - 4^x} = \frac{3}{1 - 0} = 3$

The answer is C.

8. Evaluate each statement.

I. Because $\lim_{x\to2^-} g(x) = 1$ and $\lim_{x\to2^+} g(x) = 1$, $\lim_{x\to2} g(x) = 1$.

The statement is true.

II. $\lim_{x\to2} g(x) = 1 \neq g(2) = 3$

The statement is false.

III. g is continuous at $x = 3$.

The statement is true.

Because I and III are true, the answer is B.

9. $\lim_{x\to\infty} \frac{\sqrt{9x^4 - 5}}{5x - 3x^2} = \lim_{x\to\infty} \frac{\frac{\sqrt{9x^4 - 5}}{\sqrt{x^4}}}{\frac{5x - 3x^2}{x^2}}$

$= \lim_{x\to\infty} \frac{\sqrt{9 - \frac{5}{x^4}}}{\frac{5}{x} - 3}$

$= \frac{\sqrt{9 - 0}}{0 - 3}$

$= -1$

The answer is C.

10. Because $\lim_{x\to1^-} \frac{x - 1}{\sqrt{x} - 1} = 2$ and $\lim_{x\to1^+} \frac{x - 1}{\sqrt{x} - 1} = 2$, $\lim_{x\to1} \frac{x - 1}{\sqrt{x} - 1} = 2$.

The answer is C.

11. (a) $s(t)$ is continuous on $[1, 2]$.

$s(1) = 393.1$ and $s(2) = 378.4$

Because 382 is between $s(1)$ and $s(2)$, by the Intermediate Value Theorem there exists at least one value of t in $[1, 2]$ such that $s(t) = 382$.

3 pts:
- 2 pts: shows $s(2) < 382 < s(1)$ (places 382 in this interval)
- 1 pt: justification (appeals to the continuity of s or the Intermediate Value Theorem)

Note: Merely saying "because s is differentiable" or "because s is decreasing" would not earn the justification point.

(b) $s(t) = -4.9t^2 + 398$

$0 = -4.9t^2 + 398$

$t \approx \pm 9.012$

So, the object hits the ground after approximately 9.012 seconds.

2 pts: answer with units

(c)
$$\lim_{t\to 3}\frac{s(t) - s(3)}{t - 3}$$
$$= \lim_{t\to 3}\frac{\left(-4.9t^2 + 398\right) - \left[-4.9(3)^2 + 398\right]}{t - 3}$$
$$= \lim_{t\to 3}\frac{-4.9t^2 + 4.9(9)}{t - 3}$$
$$= \lim_{t\to 3}\frac{-4.9\left(t^2 - 9\right)}{t - 3}$$
$$= \lim_{t\to 3}\frac{-4.9\cancel{(t - 3)}(t + 3)}{\cancel{t - 3}}$$
$$= \lim_{t\to 3} -4.9(t + 3)$$
$$= -4.9(3 + 3)$$
$$= -29.4 \text{ m/sec}$$

4 pts:
- 2 pts: justification (uses factoring or L'Hôpital's Rule to evaluate the limit)
- 2 pts: answer with units

Note: If applying L'Hôpital's Rule, be sure to first establish the indeterminate form $\frac{0}{0}$.

12. (a) $\lim_{x\to 0} f(x) = \lim_{x\to 0} \dfrac{10}{1 + \frac{1}{4}e^{-x}}$

$= \dfrac{10}{1 + \frac{1}{4}}$

$= \dfrac{10}{\frac{5}{4}}$

$= 8$

1 pt: answer

(b) $\lim_{x\to 0} \left[f(x) + 4\right] = \lim_{x\to 0} f(x) + \lim_{x\to 0} 4$

$= 8 + 4$

$= 12$

1 pt: answer

(c) $\lim_{x\to \infty} f(x) = \lim_{x\to \infty} \dfrac{10}{1 + \frac{1}{4}e^{-x}}$

$= \lim_{x\to \infty} \dfrac{10}{1 + \frac{1}{4e^{x}}}$

$= \dfrac{10}{1 + \frac{1}{4e^{\infty}}}$

$= \dfrac{10}{1 + 0}$

$= 10$

$\lim_{x\to -\infty} f(x) = \lim_{x\to -\infty} \dfrac{10}{1 + \frac{1}{4}e^{-x}}$

$= \dfrac{10}{1 + \frac{1}{4}e^{\infty}}$

$= \dfrac{10}{\infty}$

$= 0$

So, the horizontal asymptotes are $y = 0$ and $y = 10$.

7 pts:
- 2 pts: examines both $\lim_{x\to \infty} f(x)$ and $\lim_{x\to -\infty} f(x)$
- 3 pts: computes each limit (answers with justification)
- 2 pts: writes equations of the horizontal asymptotes (must be equations)

13. (a) $f(x) = \dfrac{x^2 + 5x + 6}{2x^2 + 7x + 3} = \dfrac{(x + 2)\cancel{(x + 3)}}{(2x + 1)\cancel{(x + 3)}}$

$= \dfrac{x + 2}{2x + 1}, x \neq -3$

$f(x)$ has discontinuities at $x = -\dfrac{1}{2}$ and $x = -3$.

2 pts: answers

(b) $\lim_{x \to -3} f(x) = \lim_{x \to -3} \dfrac{x^2 + 5x + 6}{2x^2 + 7x + 3}$

$= \lim_{x \to -3} \dfrac{(x + 2)\cancel{(x + 3)}}{(2x + 1)\cancel{(x + 3)}}$

$= \lim_{x \to -3} \dfrac{x + 2}{2x + 1}$

$= \dfrac{-3 + 2}{2(-3) + 1}$

$= \dfrac{1}{5}$

2 pts: 1 pt: justification by factoring or L'Hôpital's Rule; 1 pt: answer

(c) $f(x)$ has a vertical asymptote at $x = -\dfrac{1}{2}$.

1 pt: answer

Note: Including $x = -3$ in the answer (a removable discontinuity) would lose the answer point.

(d) $\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \dfrac{x^2 + 5x + 6}{2x^2 + 7x + 3}$

$= \lim_{x \to \infty} \dfrac{1 + \frac{5}{x} + \frac{6}{x^2}}{2 + \frac{7}{x} + \frac{3}{x^2}}$

$= \dfrac{1 + 0 + 0}{2 + 0 + 0}$

$= \dfrac{1}{2}$

$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \dfrac{x^2 + 5x + 6}{2x^2 + 7x + 3}$

$= \lim_{x \to -\infty} \dfrac{1 + \frac{5}{x} + \frac{6}{x^2}}{2 + \frac{7}{x} + \frac{3}{x^2}}$

$= \dfrac{1 + 0 + 0}{2 + 0 + 0}$

$= \dfrac{1}{2}$

$f(x)$ has a horizontal asymptote at $y = \dfrac{1}{2}$.

4 pts:
- 1 pt: examines both $\lim_{x \to \infty} f(x)$ and $\lim_{x \to -\infty} f(x)$
- 2 pts: computes each limit (correct answer for each)
- 1 pt: writes equation of the horizontal asymptote (must be an equation)

Note: Examining only one of the limits would earn 0-1-1 at most.

14. (a) $\lim_{x\to -1} f(x) = \lim_{x\to -1} e^{2x} = e^{2(-1)} = \dfrac{1}{e^2}$

1 pt: answer

(b) $f(0)$ is defined as $f(0) = e^{2(0)} = 1$.

$\lim_{x\to 0^-} f(x) = 1$ and $\lim_{x\to 0^+} f(x) = 1$,

so $\lim_{x\to 0} f(x) = 1$. Also,

$\lim_{x\to 0} f(x) = f(0) = 1$.

So, f is continuous at $x = 0$.

6 pts:
- 1 pt: finds $f(0)$
- 2 pts: finds each one-sided limit
- 1 pt: finds the limit (equates the values of the one-sided limits)
- 2 pts: indicates $f(0) = \lim_{x\to 0} f(x)$ to reach conclusion

(c)
$$\begin{aligned}\lim_{x\to -\infty} f(x) &= \lim_{x\to -\infty} e^{2x}\\ &= e^{2(-\infty)}\\ &= \frac{1}{e^{\infty}}\\ &= \frac{1}{\infty}\\ &= 0\end{aligned}$$

2 pts: answer

15. (a)
$$\begin{aligned}\lim_{x\to 1}\left[f(x) + 4\right] &= \lim_{x\to 1} f(x) + \lim_{x\to 1} 4\\ &= 2 + 4\\ &= 6\end{aligned}$$

1 pt: answer

(b) $\lim_{x\to 3^-} \dfrac{5}{g(x)} = \dfrac{5}{1} = 5$

1 pt: answer

(c) $\lim_{x\to 2}\left[f(x)\cdot g(x)\right] = 2\cdot 0 = 0$

2 pts:
- 1 pt: justification [indicates the values of both $f(2)$ and $g(2)$; writing $2 \cdot 0$ is sufficient]
- 1 pt: answer

(d)
$$\begin{aligned}\lim_{x\to 3}\frac{f(x)}{g(x) - 1} &= \lim_{x\to 3}\frac{(-2x + 6)}{(x - 2) - 1}\\ &= \lim_{x\to 3}\frac{-2x + 6}{x - 3}\\ &= \lim_{x\to 3}\frac{-2\cancel{(x - 3)}}{\cancel{(x - 3)}}\\ &= -2\end{aligned}$$

5 pts:
- 2 pts: finds linear equations for f and g on $[2, 3]$
- 2 pts: justification of limit by factoring or L'Hôpital's Rule
- 1 pt: answer

Note: The equations for f and g must both be correct to be eligible for the last three points (i.e., the limit must yield the indeterminant form $0/0$).

16. (a) Because $T(x)$ is continuous on $[0, 10)$,

$\lim_{x \to 4} T(x) = T(4) = 172.$

2 pts: 1 pt: answer
1 pt: justification (appeals to the continuity of T)

(b) $$\frac{T(8) - T(3)}{8 - 3} = \frac{164 - 174}{8 - 3} = \frac{-10}{5} = -2$$

The average rate of change is −2°F per minute.

2 pts: 1 pt: justification (evidence of a difference quotient)
1 pt: answer with units

(c) $T(x)$ is continuous and when $x = 6$, $T(x) > 166.5°$ and when $x = 8$, $T(x) < 166.5°$.

So, the shortest interval is $(6, 8)$.

3 pts: 1 pt: shows $T(8) < 166.5 < T(6)$ (places 166.5 in this interval)
1 pt: answer (an open or closed interval would generally be accepted here)
1 pt: justification (appeals to the continuity of T or the Intermediate Value Theorem)

Note: Merely stating "because T is differentiable" or "because T is decreasing" would not earn the justification point.

(d) Because $T(x)$ is continuous, the average rate of change for $6 \le x \le 9$ is

$$\frac{T(9) - T(6)}{9 - 6} = \frac{162 - 168}{9 - 6} = \frac{-6}{3} = -2.$$

So, the tangent line at $x = 8$ has a slope of about -2.

2 pts: 1 pt: justification (evidence of a difference quotient on $[6, 9]$)
1 pt: answer

17. (a) $f(x) = ax^2 + x - b$ $\qquad f(x) = ax + b$

$f(2) = a(2)^2 + (2) - b \qquad f(2) = a(2) + b$

$= 4a + 2 - b \qquad = 2a + b$

f is continuous at $x = 2$ when $4a + 2 - b = 2a + b$.

$f(x) = ax + b \qquad f(x) = 2ax - 7$

$f(5) = a(5) + b \qquad f(5) = 2a(5) - 7$

$= 5a + b \qquad = 10a - 7$

f is continuous at $x = 5$ when $5a + b = 10a - 7$.

$$4a + 2 - b = 2a + b \Rightarrow 2a - 2b = -2$$
$$5a + b = 10a - 7 \Rightarrow -5a + b = -7$$

Multiply both sides of the second equation by 2.

$$\begin{aligned} 2a - 2b &= -2 \\ -10a + 2b &= -14 \\ \hline -8a &= -16 \\ a &= 2 \end{aligned}$$

When $a = 2, b = 5(2) - 7 = 10 - 7 = 3$.

So, f is continuous when $a = 2$ and $b = 3$.

5 pts:
- 3 pts: equates $4a + 2 - b$ and $2a + b$ (equates the one-sided limits at $x = 2$) and equates $5a + b$ and $10a - 7$ (equates the one-sided limits at $x = 5$)
- 2 pts: answers (solves the system of equations to determine a and b)

(b) $\lim_{x\to 3} f(x) = \lim_{x\to 3} (2x + 3) = 2(3) + 3 = 9$

2 pts: answer from using values of a and b from part (a) and the correct piece of f

Note: Incorrect values of a and b may be imported from part (a) as long as a is nonzero (i.e., an answer consistent with values for part (a) can generally earn these two points).

(c)
$$\begin{aligned} \lim_{x\to 1} g(x) &= \lim_{x\to 1} \frac{f(x)}{x - 1} \\ &= \lim_{x\to 1} \frac{2x^2 + x - 3}{x - 1} \\ &= \lim_{x\to 1} \frac{(2x + 3)(x - 1)}{x - 1} \\ &= \lim_{x\to 1} (2x + 3) \\ &= 2(1) + 3 \\ &= 5 \end{aligned}$$

2 pts:
- 1 pt: justification (by factoring or L'Hôpital's Rule)
- 1 pt: answer

Note: To be eligible for these points, the limit must yield the indeterminant form $\frac{0}{0}$ using the values of a and b from part (a) (i.e., incorrect values of a and b generally cannot be imported here).