

AP<sup>®</sup> Exam Practice Questions for Chapter 9

$$1. \quad y(t) = 20t - \frac{5}{2}t^2 \quad x(t) = 2t^3 - 12t^2$$

$$y'(t) = 20 - 5t \quad x'(t) = 6t^2 - 24t$$

$$\frac{dy}{dx} = \frac{y'(t)}{x'(t)}$$

$$= \frac{20 - 5t}{6t^2 - 24t}$$

$$0 = \frac{20 - 5t}{6t^2 - 24t}$$

$$0 = 20 - 5t$$

$$5t = 20$$

$$t = 4$$

$$x(4) = 2(4)^3 - 12(4)^2 = -64$$

$$y(4) = 20(4) - \frac{5}{2}(4)^2 = 40$$

The particle is at rest when  $t = 4$  at the point  $(-64, 40)$ .

So, the answer is B.

$$2. \quad A = 2 \cdot \frac{1}{2} \int_0^\pi (\sin^2 \theta)^2 d\theta = \int_0^\pi \sin^4 \theta d\theta$$

So, the answer is C.

$$3. \quad x = r \cos \theta \quad y = r \sin \theta$$

$$= \theta \cos \theta \quad = \theta \sin \theta$$

$$\frac{dx}{d\theta} = -\theta \sin \theta + \cos \theta \quad \frac{dy}{d\theta} = \theta \cos \theta + \sin \theta$$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\theta \cos \theta + \sin \theta}{-\theta \sin \theta + \cos \theta}$$

$$\text{At } \theta = -\frac{\pi}{2},$$

$$\frac{dy}{dx} = \frac{-\frac{\pi}{2} \cos\left(-\frac{\pi}{2}\right) + \sin\left(-\frac{\pi}{2}\right)}{\frac{\pi}{2} \sin\left(-\frac{\pi}{2}\right) + \cos\left(-\frac{\pi}{2}\right)} = \frac{-1}{-\frac{\pi}{2}} = \frac{2}{\pi}$$

So, the answer is B.

$$6. \quad \mathbf{v}(t) = \langle x'(t), y'(t) \rangle = \left\langle 3t^2 - \frac{1}{2}, 3(3t - 2)(3) \right\rangle = \left\langle 3t^2 - \frac{1}{2}, 9(3t - 2) \right\rangle$$

$$\mathbf{a}(t) = \langle 6t, 18(3t - 2)(3) \rangle = \langle 6t, 54(3t - 2) \rangle$$

$$\mathbf{a}(1) = \langle 6(1), 54(3 \cdot 1 - 2) \rangle = \langle 6, 54 \rangle$$

So, the answer is D.

$$7. \quad \text{At } t = 4,$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{y'(t)}{x'(t)} = \frac{2e^{-4(t)} + 3}{t \cos t} = \frac{2e^{-4(4)} + 3}{4 \cos 4} \approx -1.147.$$

So, the answer is B.

$$4. \quad x = 2t^2 + 3t \quad y = t^3 + 4t^2$$

$$\frac{dx}{dt} = 4t + 3 \quad \frac{dy}{dt} = 3t^2 + 8t$$

$$\text{At } t = 2,$$

$$\frac{dy}{dx} = \frac{dx/dt}{dy/dt} = \frac{3t^2 + 8t}{4t + 3} = \frac{3(2)^2 + 8(2)}{4(2) + 3} = \frac{28}{11}.$$

$$\text{At } t = 2, x(2) = 2(2)^2 + 3(2) = 14 \text{ and}$$

$$y(2) = (2)^3 + 4(2)^2 = 24.$$

An equation of the tangent line is

$$y - 24 = \frac{28}{11}(x - 14).$$

So, the answer is B.

$$5. \quad x = r \cos \theta$$

$$= (3 + 5 \sin \theta)(\cos \theta)$$

$$= 3 \cos \theta + 5 \sin \theta \cos \theta$$

$$\frac{dx}{d\theta} = -3 \sin \theta + 5 \sin \theta (-\sin \theta) + 5(\cos \theta)(\cos \theta)$$

$$= -3 \sin \theta - 5 \sin^2 \theta + 5 \cos^2 \theta$$

$$y = r \sin \theta$$

$$= (3 + 5 \sin \theta)(\sin \theta)$$

$$= 3 \sin \theta + 5 \sin^2 \theta$$

$$\frac{dy}{d\theta} = 3 \cos \theta + 10 \sin \theta \cos \theta$$

$$\text{At } \theta = 0,$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{3 \cos \theta + 10 \sin \theta \cos \theta}{-3 \sin \theta - 5 \sin^2 \theta + 5 \cos^2 \theta}$$

$$= \frac{3(1) + 0}{0 - 0 + 5(1)} = \frac{3}{5}.$$

So, the answer is B.

8. (a) At
- $t = 3$
- , the speed is

$$\begin{aligned}\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} &= \sqrt{\left(e^{-t^2+1} - 2\right)^2 + \left(3\sqrt{25-t^2}\right)^2} \\ &= \sqrt{\left(e^{-(3)^2+1} - 2\right)^2 + \left(3\sqrt{25-(3)^2}\right)^2} \approx 12.165.\end{aligned}$$

$$\begin{aligned}\text{(b)} \quad \int_0^5 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt &= \int_0^5 \sqrt{\left(e^{-t^2+1} - 2\right)^2 + \left(3\sqrt{25-t^2}\right)^2} dt \\ &\approx 59.725\end{aligned}$$

$$\begin{aligned}\text{(c)} \quad y(3) &= y(0) + \int_0^3 \frac{dy}{dt} dt = 3 + \int_0^3 3\sqrt{25-t^2} dt \\ &= 3 + 3 \left[ \frac{1}{2} \left( t\sqrt{25-t^2} + 25 \arcsin \frac{t}{5} \right) \right]_0^3 \approx 45.131\end{aligned}$$

$$\text{(d)} \quad \frac{dy}{dx} = \frac{3\sqrt{25-t^2}}{e^{-t^2+1} - 2} \text{ is undefined when}$$

$$e^{-t^2+1} - 2 = 0$$

$$e^{-t^2+1} = 2$$

$$-t^2 + 1 = \ln 2$$

$$-t^2 = \ln 2 - 1$$

$$t = \pm\sqrt{1 - \ln 2}$$

$$t \approx 0.5539.$$

$$\mathbf{v}(t) = \langle x'(t), y'(t) \rangle = \langle e^{-t^2+1} - 2, 3\sqrt{25-t^2} \rangle$$

$$\mathbf{a}(t) = \left\langle -2te^{-t^2+1}, 3\left(\frac{1}{2}\right)(25-t^2)^{-1/2}(-2t) \right\rangle = \left\langle -2te^{-t^2+1}, -\frac{3t}{\sqrt{25-t^2}} \right\rangle$$

$$\begin{aligned}\mathbf{a}(\sqrt{1-\ln 2}) &= \left\langle -2\sqrt{1-\ln 2}e^{-(\sqrt{1-\ln 2})^2+1}, \frac{-3\sqrt{1-\ln 2}}{\sqrt{25-(\sqrt{1-\ln 2})^2}} \right\rangle \\ &\approx \langle -2.216, -0.334 \rangle\end{aligned}$$

$$\text{At } t = \sqrt{1-\ln 2} \approx 0.5539, \mathbf{a}(t) = \langle -2.216, -0.334 \rangle.$$

2 pts:  $\begin{cases} 1 \text{ pt: expression for speed} \\ 1 \text{ pt: answer} \end{cases}$

2 pts:  $\begin{cases} 1 \text{ pt: integral (arc-length)} \\ 1 \text{ pt: answer} \end{cases}$

Reminder: Write down the appropriate definite integral before numerically approximating it on your calculator. No additional work is needed.

3 pts:  $\begin{cases} 1 \text{ pt: integral} \\ 1 \text{ pt: uses initial condition} \\ 1 \text{ pt: answer} \end{cases}$

Reminder: Write down the appropriate definite integral before numerically approximating it on your calculator. No additional work is needed.

2 pts:  $\begin{cases} 1 \text{ pt: finds where } dx/dt = 0 \\ 1 \text{ pt: answer} \end{cases}$

Note: Stating

$$\mathbf{a}(0.5539) \approx \langle x''(0.5539), y''(0.5539) \rangle$$

is sufficient justification. The components of this vector can then be approximated on your calculator using the given  $x'(t)$  and  $y'(t)$  without showing further work.

Reminders: Round each answer to at least three decimal places to receive credit on the exam. Use more than three decimal places in the intermediate step of finding the  $t$ -value in part (d).

Use “ $\approx$ ” rather than an equal sign in presenting these approximations from your calculator. Because these are approximations, a point may be deducted if an equal sign is used.

$$9. (a) \quad \frac{dx}{dt} = \ln[5 + (1+t)^3] \quad \frac{dy}{dt} = 4t - 3t^2$$

$$\frac{d^2x}{dt^2} = \frac{1}{5 + (1+t)^3} \cdot 3(1+t)^2 \quad \frac{d^2y}{dt^2} = 4 - 6t$$

$$= \frac{3(1+t)^2}{5 + (1+t)^3}$$

$$\mathbf{a}(t) = \left\langle \frac{d^2x}{dt^2}, \frac{d^2y}{dt^2} \right\rangle = \left\langle \frac{3(1+t)^2}{5 + (1+t)^3}, 4 - 6t \right\rangle$$

$$\mathbf{a}(2) = \left\langle \frac{3(1+2)^2}{5 + (1+2)^3}, 4 - 6(2) \right\rangle = \left\langle \frac{27}{32}, -8 \right\rangle$$

$$\text{Speed} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

$$= \sqrt{\left(\ln[5 + (1+t)^3]\right)^2 + (4t - 3t^2)^2}$$

At  $t = 2$ , the speed is

$$\sqrt{\left(\ln[5 + (1+2)^3]\right)^2 + [4 \cdot 2 - 3(2)^2]^2} \approx 5.2926.$$

$$(b) \quad x(0) + \int_0^3 \left(\frac{dx}{dt}\right) dt = 4 + \int_0^3 \ln[5 + (1+t)^3] dt$$

$$\approx 4 + 9.0279 = 13.0279$$

$$(c) \quad \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4t - 3t^2}{\ln[5 + (1+t)^3]}$$

$$\text{At } t = 3, \quad \frac{dy}{dx} = \frac{4(3) - 3(3)^2}{\ln[5 + (1+3)^3]} = -\frac{15}{\ln 69} \approx -3.5427.$$

So, an equation of the tangent line is

$$y - 2 = -3.5427(x - 13.0279)$$

$$y = -3.5427x + 48.1535.$$

$$(d) \quad dy/dt = 4t - 3t^2 = 0 \Rightarrow t = 0, 4/3$$

$$dx/dt = \ln[5 + (1+t)^3] = 0 \Rightarrow t \approx -2.587$$

Because there is no  $t$ -value for which both  $dy/dt$  and  $dx/dt$  equal 0, the particle is never at rest.

$$2 \text{ pts: } \begin{cases} 1 \text{ pt: finds acceleration vector at } t = 2 \\ 1 \text{ pt: finds speed at } t = 2 \end{cases}$$

Notes: Stating  $\mathbf{a}(2) = \langle x''(2), y''(2) \rangle$  is sufficient justification. The components of this vector can then be approximated on your calculator using the given  $x'(t)$  and  $y'(t)$  without showing further work.

Writing Speed =  $\sqrt{(x'(2))^2 + (y'(2))^2}$  is sufficient justification. This value can then be approximated on your calculator without showing further work.

Reminders: Round each answer to at least three decimal places to receive credit on the exam.

Use “ $\approx$ ” rather than an equal sign in presenting these approximations from your calculator. Because these are approximations, a point may be deducted if an equal sign is used.

$$2 \text{ pts: } \begin{cases} 1 \text{ pt: integral} \\ 1 \text{ pt: answer} \end{cases}$$

Reminders: Write down the appropriate definite integral before numerically approximating it on your calculator. No additional work is needed.

Round each answer to at least three decimal places to receive credit on the exam.

$$3 \text{ pts: } \begin{cases} 1 \text{ pt: considers } \frac{dy}{dx} \\ 1 \text{ pt: slope at } P \\ 1 \text{ pt: equation of tangent line at } P \end{cases}$$

Reminder: Round each answer to at least three decimal places to receive credit on the exam. Use more than three decimal places when approximating the slope at  $t = 3$  in the intermediate step.

$$2 \text{ pts: } \begin{cases} 1 \text{ pt: considers where } dx/dt = 0 \text{ and } dy/dt = 0 \\ 1 \text{ pt: answer with reason} \end{cases}$$

$$\begin{aligned}
 10. (a) \quad A &= \frac{1}{2} \int_{\pi/2}^{\pi} r^2 d\theta \\
 &= \frac{1}{2} \int_{\pi/2}^{\pi} (2\theta + \cos \theta)^2 d\theta \\
 &= \frac{1}{2} \int_{\pi/2}^{\pi} (4\theta^2 + 4\theta \cos \theta + \cos^2 \theta) d\theta \\
 &= \frac{1}{2} \left[ \frac{4}{3} \theta^3 + 4(\cos \theta + \theta \sin \theta) + \frac{1}{2}(\theta + \sin \theta \cos \theta) \right]_{\pi/2}^{\pi} \\
 &\approx \frac{1}{2}(38.9125 - 12.2363) \\
 &= 13.3381
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad y &= r \sin \theta \\
 &= (2\theta + \cos \theta) \sin \theta \\
 &= 2\theta \sin \theta + \sin \theta \cos \theta = 1 \\
 &\qquad\qquad\qquad \theta \approx 2.93575 \text{ radians} \\
 x &= r \cos \theta \\
 &= (2\theta + \cos \theta) \cos \theta \\
 &= 2\theta \cos \theta + \cos^2 \theta \\
 x(2.93575) &= 2(2.93575) \cos(2.93575) + \cos^2(2.93575) \\
 &\approx -4.790
 \end{aligned}$$

$$\begin{aligned}
 (c) \quad x &= r \cos \theta \\
 &= (2\theta + \cos \theta) \cos \theta \\
 &= 2\theta \cos \theta + \cos^2 \theta \\
 \frac{dx}{dt} &= 2\theta(-\sin \theta) \frac{d\theta}{dt} + 2 \frac{d\theta}{dt} \cdot \cos \theta \\
 &\qquad\qquad\qquad + 2 \cos \theta \cdot (-\sin \theta) \frac{d\theta}{dt} \\
 \text{When } \theta &= \frac{3\pi}{4} \text{ and } \frac{d\theta}{dt} = 3, \\
 \frac{dx}{dt} &= 2\left(\frac{3\pi}{4}\right)\left(-\sin \frac{3\pi}{4}\right)(3) + 2(3)\left(\cos \frac{3\pi}{4}\right) \\
 &\qquad\qquad\qquad + 2\left(\cos \frac{3\pi}{4}\right)\left(-\sin \frac{3\pi}{4}\right)(3) \\
 &\approx -11.239.
 \end{aligned}$$

So, the particle is moving to the left on the rectangular coordinate system.

2 pts:  $\begin{cases} 1 \text{ pt: integral} \\ 1 \text{ pt: answer} \end{cases}$

Reminders: Write down the appropriate definite integral before numerically approximating it on your calculator. No additional work is needed.

Round the answer to at least three decimal places to receive credit on the exam.

Use “ $\approx$ ” rather than an equal sign in presenting the approximation from your calculator. Because this is an approximation, a point may be deducted if an equal sign is used.

3 pts:  $\begin{cases} 1 \text{ pt: finds } y(\theta) \\ 1 \text{ pt: finds } \theta \text{ for which } y = 1 \\ 1 \text{ pt: finds } x(\theta) \text{ and computes } x(2.93575) \end{cases}$

Reminders: Write down the appropriate equation before numerically approximating its solution on your calculator. No additional work is needed.

Round the answer to at least three decimal places to receive credit on the exam. Round  $\theta$  in the intermediate step to more than three decimal places.

Use “ $\approx$ ” rather than an equal sign in presenting the approximation from your calculator. Because this is an approximation, a point may be deducted if an equal sign is used.

4 pts:  $\begin{cases} 2 \text{ pts: Chain Rule with respect to } t \\ \quad \quad \quad [ \text{computes } (dx/d\theta) \cdot (d\theta/dt) ] \\ 1 \text{ pt: answer} \\ 1 \text{ pt: interprets answer} \end{cases}$

Reminders: Round the answer to at least three decimal places to receive credit on the exam.

Use “ $\approx$ ” rather than an equal sign in presenting the approximation from your calculator. Because this is an approximation, a point may be deducted if an equal sign is used.

$$\begin{aligned}
 11. (a) \quad A &= 3\pi + 2 \cdot \frac{1}{2} \int_{\pi/3}^{\pi} (2 + 2 \cos \theta)^2 d\theta \\
 &= 3\pi + \int_{\pi/3}^{\pi} (4 + 8 \cos \theta + 4 \cos^2 \theta) d\theta \\
 &= 3\pi + \left[ 4\theta + 8 \sin \theta + 4 \cdot \frac{1}{2} (\theta + \sin \theta \cos \theta) \right]_{\pi/3}^{\pi} \\
 &= 3\pi + \left[ (4\pi + 0 + 2\pi) - \left( \frac{4\pi}{3} + 4\sqrt{3} + \frac{2\pi}{3} + \frac{\sqrt{3}}{2} \right) \right] \\
 &\approx 14.197
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad \frac{dr}{dt} &= \frac{dr}{d\theta} \\
 \frac{dr}{dt} \cdot \frac{d\theta}{dr} &= \frac{dr}{d\theta} \cdot \frac{d\theta}{dr} \\
 \frac{d\theta}{dt} &= 1
 \end{aligned}$$

$$r = 2 + 2 \cos \theta$$

$$\frac{dr}{dt} = -2 \sin \theta \frac{d\theta}{dt} = -2 \sin \theta(1) = -2 \sin \theta$$

$$\text{At } \theta = \frac{\pi}{3}, \frac{dr}{dt} = -2 \sin \frac{\pi}{3} = -2 \left( \frac{\sqrt{3}}{2} \right) = -\sqrt{3} \approx -1.732.$$

So, the particle is moving closer to the origin of the polar coordinate system.

$$\begin{aligned}
 (c) \quad \frac{dx}{dt} &= \frac{dx}{d\theta} \\
 \frac{dx}{dt} \cdot \frac{d\theta}{dx} &= \frac{dx}{d\theta} \cdot \frac{d\theta}{dx} \\
 \frac{d\theta}{dt} &= 1
 \end{aligned}$$

$$x = r \cos \theta$$

$$\begin{aligned}
 \frac{dx}{dt} &= r(-\sin \theta) \frac{d\theta}{dt} + \cos \theta \frac{dr}{dt} \\
 &= (2 + 2 \cos \theta)(-\sin \theta) \frac{d\theta}{dt} + \cos \theta \frac{dr}{dt}
 \end{aligned}$$

$$\text{When } \theta = \frac{\pi}{3}, \frac{d\theta}{dt} = 1, \text{ and } \frac{dr}{dt} = -\sqrt{3},$$

$$\begin{aligned}
 \frac{dx}{dt} &= \left( 2 + 2 \cos \frac{\pi}{3} \right) \left( -\sin \frac{\pi}{3} \right) (1) + \cos \left( \frac{\pi}{3} \right) (-\sqrt{3}) \\
 &= (3) \left( -\frac{\sqrt{3}}{2} \right) + \left( \frac{1}{2} \right) (-\sqrt{3}) = -2\sqrt{3} \approx -3.4641
 \end{aligned}$$

So, the particle is moving to the left on the rectangular coordinate system.

3 pts:  $\begin{cases} 1 \text{ pt: integrand and constant} \\ 1 \text{ pt: limits of integration} \\ 1 \text{ pt: answer} \end{cases}$

Reminders: Write down the appropriate definite integral before numerically approximating it on your calculator. No further work is needed.

Round the answer to at least three decimal places to receive credit on the exam.

Use “ $\approx$ ” rather than an equal sign in presenting the approximation from your calculator. Because this is an approximation, a point may be deducted if an equal sign is used.

3 pts:  $\begin{cases} 1 \text{ pt: computes } \frac{dr}{dt} = \frac{d\theta}{dt} \\ 1 \text{ pt: answer} \\ 1 \text{ pt: interpretation} \end{cases}$

Reminder: If you choose to give a decimal approximation here, round the answer to at least three decimal places to receive credit on the exam.

3 pts:  $\begin{cases} 1 \text{ pt: represents } x \text{ in terms of } \theta \\ 1 \text{ pt: computes } \frac{dx}{dt} = \frac{dx}{d\theta} \\ 1 \text{ pt: answer and interpretation} \end{cases}$

12. (a)  $A = \frac{1}{2} \int_0^{\pi/6} (1 - 2 \sin \theta)^2 d\theta$

(b)  $x = r \cos \theta = (1 - 2 \sin \theta) \cos \theta = \cos \theta - 2 \sin \theta \cos \theta$

$$\begin{aligned} \frac{dx}{dt} &= -\sin \theta \frac{d\theta}{dt} - 2[\sin \theta(-\sin \theta) + \cos \theta(\cos \theta)] \frac{d\theta}{dt} \\ &= -\sin \theta \frac{d\theta}{dt} - 2(-\sin^2 \theta + \cos^2 \theta) \frac{d\theta}{dt} \\ &= (2 \sin^2 \theta - 2 \cos^2 \theta - \sin \theta) \frac{d\theta}{dt} \end{aligned}$$

$y = r \sin \theta = (1 - 2 \sin \theta)(\sin \theta) = \sin \theta - 2 \sin^2 \theta$

$$\frac{dy}{dt} = (\cos \theta - 4 \sin \theta \cos \theta) \frac{d\theta}{dt}$$

(c)  $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\cos \theta - 4 \sin \theta \cos \theta}{2 \sin^2 \theta - 2 \cos^2 \theta - \sin \theta}$

When  $\theta = \pi$ ,

$$\begin{aligned} \frac{dy}{dx} &= \frac{\cos \pi - 4 \sin \pi \cos \pi}{2 \sin^2 \pi - 2 \cos^2 \pi - \sin \pi} \\ &= \frac{-1 - 0}{0 - 2(-1)^2 - 0} \\ &= \frac{1}{2} \end{aligned}$$

$x(\theta) = \cos \theta - 2 \sin \theta \cos \theta$

$x(\pi) = \cos \pi - 2 \sin \pi \cos \pi = -1$

$y(\theta) = \sin \theta - 2 \sin^2 \theta$

$y(\pi) = \sin \pi - 2 \sin^2 \pi = 0$

So, an equation of the tangent line is

$$y - 0 = \frac{1}{2}(x + 1)$$

$$y = \frac{1}{2}x + \frac{1}{2}$$

3 pts:  $\begin{cases} 1 \text{ pt: integrand and constant factor} \\ 2 \text{ pts: limits of integration} \end{cases}$

3 pts:  $\begin{cases} 1 \text{ pt: writes } x \text{ and } y \text{ as functions of } \theta \\ 1 \text{ pt: finds } \frac{dx}{dt} = \frac{dx}{d\theta} \cdot \frac{d\theta}{dt} \\ 1 \text{ pt: finds } \frac{dy}{dt} = \frac{dy}{d\theta} \cdot \frac{d\theta}{dt} \end{cases}$

Reminder: You do not need to simplify these derivatives.

3 pts:  $\begin{cases} 1 \text{ pt: writes an expression for } dy/dx \text{ in terms of } \theta \\ 1 \text{ pt: finds values for } x \text{ and } y \text{ when } \theta = \pi \\ 1 \text{ pt: equation of tangent line} \end{cases}$

13. (a)  $\frac{dx}{dt}$  is positive at point  $B$  because the particle is moving to the right.  
 $\frac{dy}{dt}$  is positive at point  $B$  because the particle is moving upward.

2 pts:  $\begin{cases} 1 \text{ pt: answer and explanation for the sign of } dx/dt \\ 1 \text{ pt: answer and explanation for the sign of } dy/dt \end{cases}$

- (b) Find  $t$  when  $\frac{dy}{dt} = 0$  or undefined.

$$\frac{dy}{dt} = \frac{-2t^2}{(t^3 - \sqrt{27})^{1/3}} = 0 \text{ when } -2t^2 = 0 \Rightarrow t = 0 \text{ is}$$

undefined when

$$(t^3 - \sqrt{27})^{1/3} = 0$$

$$t^3 = 27^{1/2}$$

$$t = 27^{1/6}.$$

Because there is a cusp at point  $C$ ,  $\frac{dy}{dt}$  is undefined when  $t = 27^{1/6}$ .

2 pts:  $\begin{cases} 1 \text{ pt: reasoning [identifies that } dy/dt \text{ is undefined at } C \text{ (cusp)]} \\ 1 \text{ pt: answer (finds the } t\text{-value where } dy/dt \text{ is undefined)} \end{cases}$

$$\begin{aligned} \text{(c) } y(t) &= \int_0^{\sqrt{3}} y'(t) dt \\ &= -2 \int_0^{\sqrt{3}} \frac{t^2}{(t^3 - \sqrt{27})^{1/3}} dt \Rightarrow u = t^3 - \sqrt{27}, du = 3t^2 dt \\ &= -2 \cdot \frac{1}{3} \int_{t=0}^{t=\sqrt{3}} u^{-1/3} du \\ &= -\frac{2}{3} \left[ \frac{3}{2} u^{2/3} \right]_{t=0}^{t=\sqrt{3}} \\ &= -\frac{2}{3} \left[ \frac{3}{2} (t^3 - \sqrt{27})^{2/3} \right]_0^{\sqrt{3}} \\ &= -\left[ (\sqrt{3})^3 - (\sqrt{27})^{2/3} - (0 - \sqrt{27})^{2/3} \right] \\ &= 3 \end{aligned}$$

3 pts:  $\begin{cases} 1 \text{ pt: antiderivative (shows substitution)} \\ 1 \text{ pt: uses initial condition, } y(0) = 0 \\ 1 \text{ pt: answer} \end{cases}$

$$\begin{aligned} \text{(d) } y &= -\frac{2}{3}x + \frac{20}{3} \\ \frac{dy}{dt} &= -\frac{2}{3} \frac{dx}{dt} \end{aligned}$$

$$\text{Because } \frac{dx}{dt} \approx 1.837, \frac{dy}{dt} = -\frac{2}{3}(1.837).$$

$$\text{So, the speed is } \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{(1.837)^2 + \left[-\frac{2}{3}(1.837)\right]^2}.$$

2 pts:  $\begin{cases} 1 \text{ pt: finds } dy/dt \text{ at this point [Chain Rule: } dy/dt = (dy/dx) \cdot (dx/dt)] \\ 1 \text{ pt: finds the speed} \end{cases}$

Reminder: These approximations do not need to be simplified.