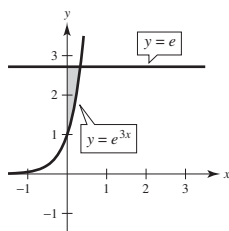


AP® Exam Practice Questions for Chapter 6

1.



$$e^{3x} = e$$

$$3x = 1$$

$$x = \frac{1}{3}$$

$$A = \int_0^{1/3} (e - e^{3x}) dx$$

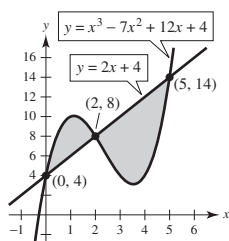
$$= \left[ex - \frac{1}{3} e^{3x} \right]_0^{1/3}$$

$$= \left(e \cdot \frac{1}{3} - \frac{1}{3} e \right) - \left(0 - \frac{1}{3} \right)$$

$$= \frac{1}{3}$$

So, the answer is A.

2.



$$x^3 - 7x^2 + 12x + 4 = 2x + 4$$

$$x^3 - 7x^2 + 10x = 0$$

$$x(x^2 - 7x + 10) = 0$$

$$x(x - 5)(x - 2) = 0$$

$$x = 0, 2, 5$$

$$A = \int_0^2 [(x^3 - 7x^2 + 12x + 4) - (2x + 4)] dx + \int_2^5 [(2x + 4) - (x^3 - 7x^2 + 12x + 4)] dx$$

$$= \int_0^2 (x^3 - 7x^2 + 10x) dx + \int_2^5 (-x^3 + 7x^2 - 10x) dx$$

$$= \left[\frac{1}{4} x^4 - \frac{7}{3} x^3 + 5x^2 \right]_0^2 + \left[-\frac{1}{4} x^4 + \frac{7}{3} x^3 - 5x^2 \right]_2^5$$

$$= \left(4 - \frac{56}{3} + 20 \right) + \left(-\frac{625}{4} + \frac{875}{3} - 125 \right) - \left(-4 + \frac{56}{3} - 20 \right)$$

$$= \frac{253}{12}$$

So, the answer is C.

$$\begin{aligned}
 3. \quad A &= \int_0^{\pi/4} (4 \cos x - 4 \sin x) dx \\
 &= [4 \sin x + 4 \cos x]_0^{\pi/4} \\
 &= \left[4 \left(\frac{\sqrt{2}}{2} \right) + 4 \left(\frac{\sqrt{2}}{2} \right) \right] - [0 + 4(1)] \\
 &= 4\sqrt{2} - 4 \\
 &= 4(\sqrt{2} - 1)
 \end{aligned}$$

So, the answer is A.

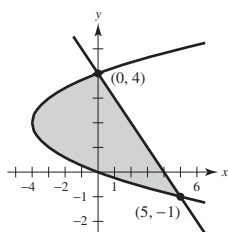
$$4. \quad 4 \sin x = 2$$

$$\begin{aligned}
 \sin x &= \frac{1}{2} \\
 x &= \frac{\pi}{6}
 \end{aligned}$$

$$\begin{aligned}
 A &= \int_0^{\pi/6} (2 - 4 \sin x) dx \\
 &= [2x + 4 \cos x]_0^{\pi/6} \\
 &= \left(\frac{\pi}{3} + 2\sqrt{3} \right) - (0 + 4) \\
 &= \frac{\pi}{3} + 2\sqrt{3} - 4
 \end{aligned}$$

So, the answer is B.

5.



Write both equations in terms of y and find the points of intersection.

$$y = -x + 4 \Rightarrow x = 4 - y$$

$$y^2 - 4y = 4 - y$$

$$y^2 - 3y - 4 = 0$$

$$(y - 4)(y + 1) = 0$$

$$y = -1, 4$$

$$\begin{aligned}
 A &= \int_{-1}^4 [(4 - y) - (y^2 - 4y)] dy \\
 &= \int_{-1}^4 (-y^2 + 3y + 4) dy \\
 &= \left[-\frac{1}{3}y^3 + \frac{3}{2}y^2 + 4y \right]_{-1}^4 \\
 &= \left(-\frac{64}{3} + 24 + 16 \right) - \left(\frac{1}{3} + \frac{3}{2} - 4 \right) \\
 &= \frac{125}{6}
 \end{aligned}$$

So, the answer is D.

$$6. \quad y = \ln(\sec x)$$

$$\begin{aligned}
 \frac{dy}{dx} &= \frac{1}{\sec x} \cdot \sec x \tan x \\
 &= \tan x
 \end{aligned}$$

$$\begin{aligned}
 s &= \int_0^{\pi/4} \sqrt{1 + \tan^2 x} dx \\
 &= \int_0^{\pi/4} \sqrt{\sec^2 x} dx \\
 &= \int_0^{\pi/4} \sec x dx
 \end{aligned}$$

So, the answer is B.

$$7. \quad y = 4e^{0.5x}$$

$$\begin{aligned}
 \frac{dy}{dx} &= 0.5(4e^{0.5x}) \\
 &= 2e^{0.5x}
 \end{aligned}$$

$$\begin{aligned}
 s &= \int_1^4 \sqrt{1 + (2e^{0.5x})^2} dx \\
 &= \int_1^4 \sqrt{1 + 4e^x} dx
 \end{aligned}$$

So, the answer is A.

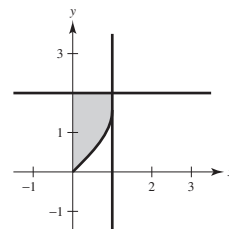
$$8. \quad y = \frac{2}{3}x^{3/2}$$

$$\frac{dy}{dx} = x^{1/2}$$

$$\begin{aligned}
 s &= \int_3^8 \sqrt{1 + (x^{1/2})^2} dx \\
 &= \int_3^8 \sqrt{1 + x} dx \\
 &= \left[\frac{2}{3}(1 + x)^{3/2} \right]_3^8 \\
 &= \frac{2}{3}(27 - 8) \\
 &= \frac{38}{3}
 \end{aligned}$$

So, the answer is B.

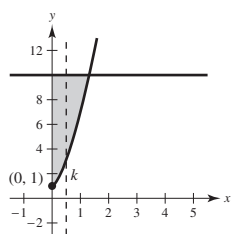
9.



$$\begin{aligned}
 V &= \int_0^1 [2 - \sin^{-1} x]^2 dx \\
 &\approx 2.184
 \end{aligned}$$

So, the answer is C.

10. (a)



Find the point of intersection of the graphs.

$$10 = 1 + 6x^{3/2}$$

$$9 = 6x^{3/2}$$

$$\frac{3}{2} = x^{3/2}$$

$$\left(\frac{3}{2}\right)^{2/3} = x$$

$$x \approx 1.3103707$$

$$\int_0^k [10 - (1 + 6x^{3/2})] dx = \int_k^{1.31} [10 - (1 + 6x^{3/2})] dx$$

or

$$y = 1 + 6x^{3/2}$$

$$y - 1 = 6x^{3/2}$$

$$\frac{y - 1}{6} = x^{3/2}$$

$$x = \left(\frac{y - 1}{6}\right)^{2/3}$$

$$\int_1^{10} \left(\frac{y - 1}{6}\right)^{2/3} dy = 2 \int_0^{1+6k^{3/2}} \left(\frac{y - 1}{6}\right)^{2/3} dy$$

(b)

$$y = 1 + 6x^{3/2}$$

$$y' = 6\left(\frac{3}{2}\right)x^{1/2} = 9x^{1/2}$$

$$1 + (y')^2 = 1 + (9x^{1/2})^2 = 1 + 81x$$

$$s = \int_0^1 \sqrt{1 + 81x} dx$$

$$= \frac{1}{81} \int_0^1 (1 + 81x)^{1/2} (81) dx$$

$$= \frac{1}{81} \left[\frac{2}{3} (1 + 81x)^{3/2} \right]_0^1$$

$$= \frac{2}{243} (82^{3/2} - 1) \approx 6.103$$

(c) $A = w \cdot h$

$$= [10 - (1 + 6x^{3/2})] \cdot 3[10 - (1 + 6x^{3/2})]$$

$$= 3(9 - 6x^{3/2})^2$$

$$V = 3 \int_0^{1.3103707} (9 - 6x^{3/2})^2 dx \approx 143.289$$

3 pts: $\begin{cases} 1 \text{ pt: integrand} \\ 2 \text{ pts: limits of integration within correct equation} \end{cases}$

Reminders: Use your calculator to find the point of intersection of the two graphs (no work needed for this).

In this intermediate step, round the x-coordinate of the intersection point to more than three decimal places to use in upcoming integrals.

3 pts: $\begin{cases} 2 \text{ pts: integral} \\ 1 \text{ pt: answer (no work needed)} \end{cases}$

Reminders: Be sure to write down the appropriate definite integral before numerically approximating it on your calculator.

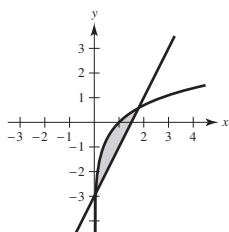
Be sure to round the answer to at least three decimal places to receive credit on the exam.

3 pts: $\begin{cases} 2 \text{ pts: integral} \\ 1 \text{ pt: answer (no work needed)} \end{cases}$

Reminders: Be sure to write down the appropriate definite integral before numerically approximating it on your calculator.

Be sure to round the answer to at least three decimal places to receive credit on the exam.

11. (a)

Rewrite the equations in terms of y .

$$y = \ln x \Rightarrow x = e^y$$

$$y = 2x - 3 \Rightarrow x = \frac{1}{2}y + \frac{3}{2}$$

The graphs intersect at $y \approx -2.888703$ and $y \approx 0.58307388$.

$$\begin{aligned} A &= \int_{-2.888703}^{0.58307388} \left[\left(\frac{1}{2}y + \frac{3}{2} \right) - e^y \right] dy \\ &= \left[\frac{1}{4}y^2 + \frac{3}{2}y - e^y \right]_{-2.888703}^{0.58307388} \\ &\approx (-0.832) - (-2.303) \\ &= 1.471 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad V &= \pi \int_{0.05564832}^{1.7915369} \left([\ln(x) + 3]^2 - [(2x - 3) + 3]^2 \right) dx \\ &\approx 18.783 \end{aligned}$$

$$\text{(c)} \quad \pi \int_{-2.888703}^{0.58307388} \left[\left(\frac{y+3}{2} \right)^2 - (e^y)^2 \right] dy$$

$$3 \text{ pts: } \begin{cases} 1 \text{ pt: integrand} \\ 1 \text{ pt: limits of integration} \\ 1 \text{ pt: answer (no work needed)} \end{cases}$$

Reminders: Use your calculator to find the points of intersection of the two graphs (no work needed for this).

In this intermediate step, round the coordinates of the intersection points to more than three decimal places to use in upcoming integrals.

$$3 \text{ pts: } \begin{cases} 2 \text{ pts: integrand} \\ 1 \text{ pt: limits of integration, constant, answer} \end{cases}$$

Reminders: Be sure to write down the appropriate definite integral before numerically approximating it on your calculator.

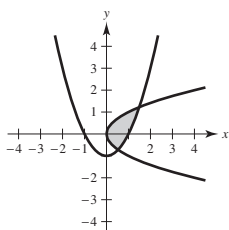
Be sure to round the answer to at least three decimal places to receive credit on the exam.

$$3 \text{ pts: } \begin{cases} 2 \text{ pts: integrand} \\ 1 \text{ pt: limits of integration and constant} \end{cases}$$

Notes: To use disks (washers), set up this integral in terms of y (using horizontal elements in R).

Setting up the volume integral in terms of x (using vertical elements in R) requires the shell method.

12. (a)



Find the points of intersection of the graphs

$$y = x^2 - 1 \Rightarrow x = \pm\sqrt{y+1}$$

$$\sqrt{y+1} = y^2$$

$$y+1 = y^4$$

$$0 = y^4 - y - 1$$

$$y \approx -0.724492, 1.2207441$$

$$A = \int_{-0.724492}^{1.2207441} (\sqrt{y+1} - y^2) dy$$

$$= \left[\frac{2}{3}(y+1)^{3/2} - \frac{1}{3}y^3 \right]_{-0.724492}^{1.2207441}$$

$$\approx 1.377$$

$$(b) \quad V = \pi \int_{-0.724492}^{1.2207441} \left[(2 - y^2)^2 - (2 - \sqrt{y+1})^2 \right] dy$$

$$\approx 11.501$$

$$(c) \quad 2\pi \int_{-0.724492}^{1.2207441} y(\sqrt{y+1} - y^2) dy \text{ or}$$

$$\pi \int_0^{0.5248886} \left[(\sqrt{x} + 1)^2 - (-\sqrt{x} + 1)^2 \right] dx$$

$$+ \pi \int_{0.5248886}^{1.4902161} \left[(\sqrt{x} + 1)^2 - (x^2)^2 \right] dx$$

3 pts: $\begin{cases} 1 \text{ pt: integrand} \\ 1 \text{ pt: limits of integration} \\ 1 \text{ pt: answer (no work needed)} \end{cases}$

Note: Because R is horizontally simple, it is strategic to set up this area integral in terms of y (using horizontal elements in R). Setting up the area integral in terms of x (using vertical elements in R) requires the sum of multiple definite integrals because R is not vertically simple.

Reminders: Use your calculator to find the points of intersection of the two graphs (no work needed for this).

In this intermediate step, round the coordinates of the intersection points to more than three decimal places to use in upcoming integrals.

Be sure to write down the appropriate definite integral before numerically approximating it on your calculator.

Be sure to round the answer to at least three decimal places to receive credit on the exam.

3 pts: $\begin{cases} 2 \text{ pts: integrand} \\ 1 \text{ pt: limits of integration, constant, answer} \end{cases}$

Notes: To use disk (washers), set up this integral in terms of y (using horizontal elements in R).

Setting up this volume integral in terms of x (using vertical elements in R), via the shell method, requires the sum of multiple definite integrals because R is not vertically simple.

Reminders: Be sure to write down the appropriate definite integral before numerically approximating it on your calculator.

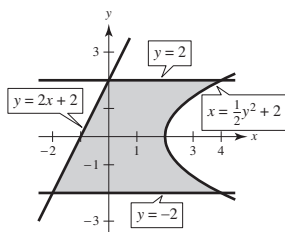
Be sure to round the answer to at least three decimal places to receive credit on the exam.

3 pts: $\begin{cases} 2 \text{ pts: integrand} \\ 1 \text{ pt: limits and constant} \end{cases}$

Notes: Because R is horizontally simple, it is strategic to set up the shell method here (using horizontal elements in R) in order to set up a *single* integral.

Setting up this volume integral in terms of x (using vertical elements in R), via the disk/washer method, requires the sum of multiple definite integrals because R is not vertically simple.

13. (a)



$$\begin{aligned}
 \text{(b) } A &= \int_{-2}^2 \left[\left(\frac{y^2}{2} + 2 \right) - \left(\frac{y-2}{2} \right) \right] dy \\
 &= \int_{-2}^2 \left(\frac{1}{2}y^2 - \frac{1}{2}y + 3 \right) dy \\
 &= \left[\frac{1}{6}y^3 - \frac{1}{4}y^2 + 3y \right]_{-2}^2 \\
 &= \left(\frac{19}{3} \right) - \left(-\frac{25}{3} \right) \\
 &= \frac{44}{3}
 \end{aligned}$$

3 pts: sketch of correct region (with boundary curves labeled)

6 pts: $\begin{cases} 2 \text{ pts: integrand} \\ 2 \text{ pts: limits} \\ 1 \text{ pt: antiderivative} \\ 1 \text{ pt: answer} \end{cases}$

Note: Because the region is horizontally simple, it is strategic to set up this area integral in terms of y (using horizontal elements of R). Setting up this area integral in terms of x (using vertical elements in R) requires the sum of multiple definite integrals because R is not vertically simple.

Reminder: The answer does *not* need to be simplified.

$$14. \text{ (a) } V = \pi \int_0^2 (\sqrt{x})^2 dx = \pi \int_0^2 x dx = \frac{\pi}{2} [x^2]_0^2 = 2\pi$$

2 pts: $\begin{cases} 1 \text{ pt: integral} \\ 1 \text{ pt: answer} \end{cases}$

$$\begin{aligned}
 \text{(b) } V &= \pi \int_0^{\sqrt{2}} [2^2 - (y^2)^2] dy = \pi \int_0^{\sqrt{2}} (4 - y^4) dy \\
 &= \pi \left[4y - \frac{1}{5}y^5 \right]_0^{\sqrt{2}} = \frac{16}{5}\pi\sqrt{2}
 \end{aligned}$$

3 pts: $\begin{cases} 2 \text{ pts: integral} \\ 1 \text{ pt: answer} \end{cases}$

Notes: To use disks (washers), set up this integral in terms of y (using horizontal elements in R).

Setting up the volume integral in terms of x (using vertical elements in R) requires the shell method.

$$\begin{aligned}
 \text{(c) } \pi \int_0^2 [(-2 - \sqrt{x})^2 - (-2)^2] dx \\
 = \pi \int_0^2 [(-2 - \sqrt{x})^2 - 4] dx
 \end{aligned}$$

2 pts: integral

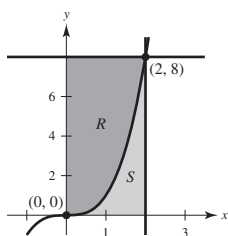
Notes: To use disks (washers), set up this integral in terms of x (using vertical elements in R).

Setting up the volume integral in terms of y (using horizontal elements in R) requires the shell method.

$$\begin{aligned}
 \text{(d) } A &= \frac{1}{2}bh = \frac{1}{2}(\sqrt{x})\left(\frac{\sqrt{3}}{2}\sqrt{x}\right) = \frac{\sqrt{3}}{4}x \\
 V &= \int_0^2 \frac{\sqrt{3}}{4}x dx = \left[\frac{\sqrt{3}}{8}x^2 \right]_0^2 = \frac{\sqrt{3}(2)^2}{8} = \frac{\sqrt{3}}{2}
 \end{aligned}$$

2 pts: $\begin{cases} 1 \text{ pt: integral} \\ 1 \text{ pt: answer} \end{cases}$

15.



$$\begin{aligned}
 \text{(a)} \quad A &= \int_0^2 (8 - x^3) \, dx \\
 &= \left[8x - \frac{1}{4}x^4 \right]_0^2 \\
 &= 16 - 4 \\
 &= 12
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad V &= 2\pi \int_0^2 x(8 - x^3) \, dx \\
 &= 2\pi \int_0^2 (8x - x^4) \, dx \\
 &= 2\pi \left[4x^2 - \frac{1}{5}x^5 \right]_0^2 \\
 &= \frac{96\pi}{5}
 \end{aligned}$$

or

$$\begin{aligned}
 V &= \pi \int_0^8 (3\sqrt{y})^2 \, dy \\
 &= \pi \int_0^8 y^{2/3} \, dy \\
 &= \pi \left[\frac{3}{5}y^{5/3} \right]_0^8 \\
 &= \frac{96\pi}{5}
 \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad A &= \frac{\pi}{2} \left(\frac{x^3}{2} \right)^2 = \frac{\pi}{8} x^6 \\
 V &= \frac{\pi}{8} \int_0^2 x^6 \, dx \\
 &= \frac{\pi}{8} \left[\frac{1}{7}x^7 \right]_0^2 \\
 &= \frac{128\pi}{56} \\
 &= \frac{16\pi}{7}
 \end{aligned}$$

3 pts: $\begin{cases} 1 \text{ pt: integrand} \\ 1 \text{ pt: limits} \\ 1 \text{ pt: answer} \end{cases}$

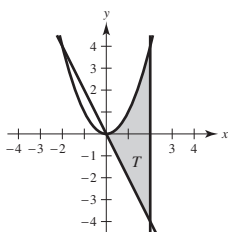
3 pts: $\begin{cases} 2 \text{ pts: integral} \\ 1 \text{ pt: answer} \end{cases}$

Notes: To use disks, set up this integral terms of y (using horizontal elements in R).

Setting up the volume integral in terms of x (using vertical elements in R) requires the shell method.

3 pts: $\begin{cases} 2 \text{ pts: integral} \\ 1 \text{ pt: answer} \end{cases}$

16.



$$\begin{aligned}
 \text{(a)} \quad A &= \int_0^2 [x^2 - (-2x)] dx \\
 &= \int_0^2 (x^2 + 2x) dx \\
 &= \left[\frac{1}{3}x^3 + x^2 \right]_0^2 \\
 &= \frac{8}{3} + 4 \\
 &= \frac{20}{3}
 \end{aligned}$$

3 pts: $\begin{cases} 1 \text{ pt: integrand} \\ 1 \text{ pt: limits} \\ 1 \text{ pt: answer} \end{cases}$

$$\begin{aligned}
 \text{(b)} \quad V &= \pi \int_0^2 [(-4 - x^2)^2 - (-4 - [-2x])^2] dx \\
 &= \pi \int_0^2 [(16 + 8x^2 + x^4) - (16 - 16x + 4x^2)] dx \\
 &= \pi \int_0^2 (x^4 + 4x^2 + 16x) dx \\
 &= \pi \left[\frac{1}{5}x^5 + \frac{4}{3}x^3 + 8x^2 \right]_0^2 \\
 &= \pi \left(\frac{32}{5} + 32 + \frac{32}{5} \right) \\
 &= \frac{736\pi}{5}
 \end{aligned}$$

3 pts: $\begin{cases} 2 \text{ pts: integral} \\ 1 \text{ pt: answer} \end{cases}$

$$\begin{aligned}
 \text{(c)} \quad P &= \sqrt{(2-2)^2 + (-4-4)^2} + \sqrt{(2-0)^2 + (-4-0)^2} \\
 &\quad + \int_0^2 \sqrt{1 + (2x)^2} dx \\
 &= 8 + 2\sqrt{5} + \int_0^2 \sqrt{1 + 4x^2} dx
 \end{aligned}$$

3 pts: $\begin{cases} 2 \text{ pts: arc length integral} \\ 1 \text{ pt: answer (sum of lengths)} \end{cases}$