

## AP® Exam Practice Questions for Chapter 4

1. Use the Trapezoidal Rule to find an approximation of
- $\int_0^4 g(x) dx$
- .

$$\begin{aligned}\int_0^4 g(x) dx &\approx \frac{4-0}{2(8)} [0 + 2(3) + 2(7) + 2(12) + 2(18) + 2(25) + 2(33) + 2(42) + 52] \\ &= \frac{1}{4}(332) = 83\end{aligned}$$

So, the answer is B.

$$\begin{aligned}2. \int_0^5 f(x) dx &= \int_0^3 f(x) dx + \int_3^5 f(x) dx \\ &= \frac{1}{2}(3+5)3 + 2(-2) \\ &= 8\end{aligned}$$

So, the answer is C.

3.  $\int \frac{4}{(x-5)^2 + 9} dx$

Let  $u = x - 5$ ,  $du = dx$ , and  $a = 3$ .

$$\begin{aligned}4 \int \frac{1}{u^2 + a^2} dx &= 4 \left[ \frac{1}{a} \arctan \frac{u}{a} + C \right] \\ &= 4 \left( \frac{1}{3} \arctan \frac{x-5}{3} + C \right) \\ &= \frac{4}{3} \tan^{-1} \frac{x-5}{3} + C\end{aligned}$$

So, the answer is A.

4.  $v(t) = 4t^3 - 4t$ ,  $0 \leq t \leq 2$

$$\begin{aligned}\frac{1}{2-0} \int_0^2 v(t) dt &= \frac{1}{2} \int_0^2 (4t^3 - 4t) dt \\ &= \frac{1}{2} [t^4 - 2t^2]_0^2 \\ &= \frac{1}{2}(16 - 8) \\ &= 4 \text{ units/sec}\end{aligned}$$

So, the answer is A.

$$\begin{aligned}8. \frac{1}{\frac{3\pi}{2} - 0} \int_0^{3\pi/2} (x + \sin x) dx &= \frac{2}{3\pi} \left[ \frac{1}{2}x^2 - \cos x \right]_0^{3\pi/2} \\ &= \frac{2}{3\pi} \left[ \left( \frac{9\pi^2}{8} - 0 \right) - (0 - 1) \right] \\ &\approx 2.568\end{aligned}$$

So, the answer is C.

$$\begin{aligned}5. g(-1) &= \int_4^{-1} f(t) dt \\ &= -\int_{-1}^4 f(t) dt \\ &= -\int_{-1}^0 f(t) dt - \int_0^3 f(t) dt - \int_3^4 f(t) dt \\ &= -\frac{1}{2}(-2) - \frac{1}{2}(4)(-2) - \frac{1}{2}(2) \\ &= 4\end{aligned}$$

So, the answer is C.

$$\begin{aligned}6. \int_b^\pi \sin x dx &= 0.4 \\ [-\cos x]_b^\pi &= 0.4 \\ -\cos \pi + \cos b &= 0.4 \\ 1 + \cos b &= 0.4 \\ \cos b &= -0.6 \\ b &\approx 2.214\end{aligned}$$

So, the answer is D.

$$\begin{aligned}7. f'(x) &= \sqrt{x^3 + 6} \\ a &= 1, b = 5 \\ f(b) &= f(a) + \int_a^b f'(x) dx \\ f(5) &= f(1) + \int_1^5 \sqrt{x^3 + 6} dx \\ &\approx 2 + 24.672 \\ &= 26.672\end{aligned}$$

So, the answer is D.

$$\begin{aligned} 9. (a) \int_0^{12} C'(t) dt &= C(12) - C(0) \\ &= 40 - 65 \\ &= -25^\circ\text{C} \end{aligned}$$

The total temperature lost from  $t = 0$  to  $t = 12$  is  $25^\circ\text{C}$ .

(b)  $\frac{1}{12} \int_0^{12} C(t) dt$  represents the average temperature for  $0 \leq t \leq 12$ .

$$\begin{aligned} \frac{1}{12} \int_0^{12} C(t) dt &= \frac{1}{12} \left[ \int_0^3 C(t) dt + \int_3^5 C(t) dt + \int_5^7 C(t) dt \right. \\ &\quad \left. + \int_7^8 C(t) dt + \int_8^{12} C(t) dt \right] \\ &= \frac{1}{12} \left[ \frac{3-0}{2}(65+57) + \frac{5-3}{2}(57+50) \right. \\ &\quad \left. + \frac{7-5}{2}(50+46) \right. \\ &\quad \left. + \frac{8-7}{2}(46+44) \right. \\ &\quad \left. + \frac{12-8}{2}(44+40) \right] \\ &\approx 49.917^\circ\text{C} \end{aligned}$$

So, the average temperature of the coffee is about  $49.917^\circ\text{C}$ .

$$\begin{aligned} (c) C'(4) &\approx \frac{C(5) - C(3)}{5 - 3} = \frac{50 - 57}{2} \\ &= -3.5 \end{aligned}$$

So, when  $t = 4$ , the temperature of the coffee is changing about  $-3.5^\circ\text{C}$  per minute.

$$\begin{aligned} (d) C(t) &= \int C'(t) dt = \int -2 \cos 0.5t dt \\ &= -2 \left( \frac{1}{0.5} \right) \int \cos(0.5t)(0.5) dt \\ &= -4(\sin 0.5t) + K \\ &= -4 \sin 0.5t + K \end{aligned}$$

Because  $C(t)$  is continuous at  $C = 12$ , use  $C(12) = 40$  to find  $K$ .

$$\begin{aligned} -4 \sin[0.5(12)] + K &= 40 \\ K &\approx 38.8823 \end{aligned}$$

$$\begin{aligned} \text{So, } C(15) &= -4 \sin(0.5 \cdot 15) + 38.8823 \\ &\approx 35.130^\circ\text{C}. \end{aligned}$$

2 pts:  $\begin{cases} 1 \text{ pt: answer} \\ 1 \text{ pt: units and interpretation of answer} \end{cases}$

Note: Could simply explain as “change in temperature from time  $t = 0$  to  $t = 12$ .”

3 pts:  $\begin{cases} 1 \text{ pt: identifies as average temperature} \\ 2 \text{ pts: answer with units and analysis} \end{cases}$

1 pt: difference quotient with units

3 pts:  $\begin{cases} 1 \text{ pt: antiderivative} \\ 1 \text{ pt: uses } C(12) = 40 \text{ to find constant of integration} \\ 1 \text{ pt: finds } C(15) \text{ with units} \end{cases}$

Reminder: Use more than three decimal places when representing the constant of integration (avoid premature rounding in this intermediate step) so that the final answer can be rounded to three decimal places. Perhaps store the value of the constant in your calculator for use in the subsequent computation.

Reminder: Round each answer to at least three decimal places to receive credit on the exam.

$$\begin{aligned}
 10. (a) \int_0^6 M(t) dt &= \int_0^6 \frac{\pi}{6} \sin \frac{\pi t}{12} dt \\
 &= 2 \int_0^6 \sin \frac{\pi - 6}{12} \left( \frac{\pi}{12} \right) dt \\
 &= 2 \left[ -\cos \frac{\pi t}{12} \right]_0^6 \\
 &= 2[0 - (-1)] \\
 &= 2
 \end{aligned}$$

So, 2 inches of snow will melt during the 6 hour period.

$$\begin{aligned}
 (b) I(t) &= S(t) - M(t) + 40 \\
 &= 0.006t^2 - 0.12t + 0.87 - \frac{\pi}{6} \sin \frac{\pi t}{12} + 40 \\
 &= 0.006t^2 - 0.12t + 40.87 - \frac{\pi}{6} \sin \frac{\pi t}{12}
 \end{aligned}$$

$$\begin{aligned}
 (c) I'(t) &= 0.012t - 0.12 - \frac{\pi}{6} \cos \frac{\pi t}{12} \left( \frac{\pi}{12} \right) \\
 I'(3) &= 0.012(3) - 0.12 - \frac{\pi^2}{72} \cos \left( \frac{\pi}{4} \right) \\
 &\approx -0.181 \text{ in./h}
 \end{aligned}$$

(d) Because  $I(t)$  is decreasing over  $[0, 6]$ , the maximum is at  $t = 0$ .  
 $I(0) = 40.87$ , so the maximum amount of snow is 40.87 inches.

$$3 \text{ pts: } \begin{cases} 1 \text{ pt: definite integral [writing } \int_0^6 M(t) dt \text{ is sufficient]} \\ 1 \text{ pt: answer (no work needed, use calculator)} \\ 1 \text{ pt: units} \end{cases}$$

$$2 \text{ pts: } \begin{cases} 1 \text{ pt: includes } S(t) - M(t) \text{ in answer} \\ 1 \text{ pt: includes the initial value, 40, in answer} \end{cases}$$

$$2 \text{ pts: } \begin{cases} 1 \text{ pt: answer [computes } I'(3), \text{ no work needed]} \\ 1 \text{ pt: units (a rate)} \end{cases}$$

$$2 \text{ pts: } \begin{cases} 1 \text{ pt: answers} \\ 1 \text{ pt: reason [identifies that } I(t) \text{ is decreasing on this interval]} \end{cases}$$

Reminder: Round each answer to at least three decimal places to receive credit on the exam.

11. (a)  $v(t) = \sin \frac{\pi t}{4}$

The particle is moving to the right on the  $t$ -intervals  $(0, 4)$  and  $(8, 9)$  because  $v(t) > 0$  on these intervals.

(b)  $\int_0^9 \sin \frac{\pi t}{4} dt$

$$\begin{aligned} \text{(c) } v'(t) &= \frac{\pi}{4} \cos \frac{\pi t}{4} \\ a(3) &= v'(3) \\ &= \frac{\pi}{4} \cos \frac{\pi(3)}{4} \end{aligned}$$

Because  $3\pi/4$  is in Quadrant II,  $\cos(3\pi/4)$  is negative. So,  $a(3) < 0$ . Because  $v(3) > 0$ , the acceleration and velocity are in opposite directions. This means that the particle is slowing down.

$$\begin{aligned} \text{(d) } s(t) &= \int v(t) dt \\ &= \int \sin \frac{\pi t}{4} dt \\ &= \frac{4}{\pi} \int \sin \frac{\pi t}{4} \left( \frac{\pi}{4} \right) dt \\ &= -\frac{4}{\pi} \cos \frac{\pi t}{4} + C \end{aligned}$$

Use  $s(0) = -4$  to find  $C$ .

$$\begin{aligned} -4 &= -\frac{4}{\pi} \cos 0 + C \\ -4 + \frac{4}{\pi} &= C \end{aligned}$$

$$\text{So, } s(t) = -\frac{4}{\pi} \cos \frac{\pi t}{4} + \frac{4 - 4\pi}{\pi}.$$

$$\text{Therefore, } s(3) = -\frac{4}{\pi} \cos \frac{3\pi}{4} + \frac{4 - 4\pi}{\pi}.$$

2 pts: answers with reason [identifies where  $v(t) > 0$ ]

Reminder: Be sure to explicitly identify each function by name. Referring to  $v(t)$  as “it,” “the function,” or “the graph” may not receive credit on the exam.

1 pt: definite integral

3 pts:  $\left\{ \begin{array}{l} 1 \text{ pt: computes } a(t) = v'(t) \text{ (Chain Rule)} \\ 1 \text{ pt: answer for } a(3) \\ 1 \text{ pt: answer (“slowing down”) with explanation} \end{array} \right.$

Reminder:  $a(3)$  does *not* need to be evaluated or simplified. Leaving the answer as  $\frac{\pi}{4} \cos \frac{3\pi}{4}$  or

$\frac{\pi}{4} \left( -\frac{\sqrt{2}}{2} \right)$  is sufficient.

3 pts:  $\left\{ \begin{array}{l} 1 \text{ pt: antiderivative} \\ 1 \text{ pt: uses initial condition to find constant of integration} \\ 1 \text{ pt: answer} \end{array} \right.$

Reminder: The answer does not need to be evaluated or simplified.

$$12. F(x) = \int_3^x f(t) dt$$

$$\begin{aligned} \text{(a)} \quad F(0) &= \int_3^0 f(t) dt \\ &= -\int_0^3 f(t) dt \\ &= -\left[\frac{1}{4}\pi(2)^2 + \frac{1}{2}(2+3)(1)\right] \\ &\approx -5.64 \end{aligned}$$

$$F'(0) = f(0) = 3$$

$$\begin{aligned} F(4) &= \int_3^4 f(t) dt \\ &= \frac{1}{2}(-1)(1) = -0.5 \end{aligned}$$

(b) The graph of  $F(x)$  does not have a minimum value because  $F'(x) = f(x)$  does not change from negative to positive at any point.

(c) Because  $F'(x) = f(x)$  changes from increasing to decreasing at  $x = 0$ , an inflection point of the graph of  $F(x)$  is  $x = 0$ .

(d) Because  $F'(2) = f(2) = 1$ , the slope of the tangent line is 1. Use  $F(2) = \int_3^2 f(t) dt = -\frac{1}{2}$  and  $m = 1$  to write the equation of the tangent line.

$$y + \frac{1}{2} = 1(x - 2)$$

$$y = x - \frac{5}{2}$$

So, the equation of the tangent line at  $F(2)$

$$\text{is } y = x - \frac{5}{2}.$$

3 pts:  $\left\{ \begin{array}{l} 1 \text{ pt: answer for } F(0) \text{ with justification} \\ \quad \text{(computes signed area)} \\ 1 \text{ pt: answer for } F'(0) \text{ with justification} \\ \quad \text{[indicates } F'(0) = f(0)] \\ 1 \text{ pt: answer for } F(4) \text{ with justification} \\ \quad \text{(computes signed area)} \end{array} \right.$

2 pts: answer with reason [identifies that  $F'(x) = f(x)$  does not change from negative to positive on this interval]

2 pts: answer with reason [identifies where  $F'(x) = f(x)$  changes from increasing to decreasing]

2 pts:  $\left\{ \begin{array}{l} 1 \text{ pt: finds } \textit{slope} \text{ of the line tangent to } F(x) \text{ at } x = 2 \\ \quad \text{[finds } F'(2) = f(2)] \\ 1 \text{ pt: finds } F(2) \text{ (signed area) and an } \textit{equation} \text{ of the} \\ \quad \text{line tangent to } F \text{ at } x = 2 \end{array} \right.$

13. (a)  $\int_0^1 f(x) dx = -5.5$

$F(1) - F(0) = -5.5$

$9 - F(0) = -5.5$

$F(0) = 14.5$

$\int_1^3 f(x) dx = -6$

$F(3) - F(1) = -6$

$F(3) - 9 = -6$

$F(3) = 3$

$\int_3^4 f(x) dx = 15.5$

$F(4) - F(3) = 15.5$

$F(4) - 3 = 15.5$

$F(4) = 18.5$

So,  $F(0) = 14.5$  and  $F(4) = 18.5$ .

- (b) Because  $F(0) > 5 > F(3)$  and  $F(x)$  is continuous on  $[0, 3]$ , there is at least one  $x$ -value in this interval where  $F(x) = 5$  by the Intermediate Value Theorem. Because  $F(3) < 5 < F(4)$  and  $F(x)$  is continuous on  $[3, 4]$ , there is at least one  $x$ -value in this interval where  $F(x) = 5$ . So,  $F(x)$  must equal 5 at least 2 times on  $[0, 4]$ .

- (c) Because  $F'(x) = f(x) > 0$  on the interval  $(3, 4)$ ,  $F$  is increasing on the interval  $(3, 4)$ .

$$4 \text{ pts: } \left\{ \begin{array}{l} 2 \text{ pts: finds } F(0) \text{ with justification} \\ \quad \left[ \text{uses } \int_0^1 f(x) dx = -5.5 \text{ and } F(1) = 9 \right] \\ 2 \text{ pts: finds } F(4) \text{ with justification} \\ \quad \left[ \text{uses } \int_1^3 f(x) dx = -6 \text{ and } F(1) = 9 \text{ to} \right. \\ \quad \text{find } F(3), \text{ and then uses } \int_3^4 f(x) dx = 15.5 \\ \quad \left. \text{and } F(3) = 3 \text{ to find } F(4) \right] \end{array} \right.$$

- 3 pts: answer with reason  $\left[ \text{appeals to values of } F(0), F(3), \right.$   
and  $F(4)$  and the continuity of  $F(x)$ ]

- 2 pts: answer with reason  $\left[ \text{appeals to where} \right.$   
 $F'(x) = f(x) > 0$ ]