AP® Exam Practice Questions for Chapter 1

1. $\lim_{x\to\pi} h(x) = \lim_{x\to\pi} 2 = 2$

The answer is B.

2. $\lim_{x\to-4^-} g(x) = \lim_{x\to-4^-} \dfrac{|x+4|}{x+4} = -1$

$\lim_{x\to-4^+} g(x) = \lim_{x\to-4^+} \dfrac{|x+4|}{x+4} = 1$

Because $\lim_{x\to-4^-} g(x) \neq \lim_{x\to-4^+}$, the limit is nonexistent.

The answer is D.

3. $\lim_{x\to\pi} \dfrac{\sin x}{x} = \dfrac{\sin \pi}{\pi} = \dfrac{0}{\pi} = 0$

The answer is A.

4. $\lim_{x\to-2} \dfrac{3x^2 + 5x + 7}{x - 4}$

$= \dfrac{3(-2)^2 + 5(-2) + 7}{(-2) - 4}$

$= \dfrac{9}{-6}$

$= -\dfrac{3}{2}$

The answer is B.

5. $\lim_{x\to5} \left[5f(x) - g(x)\right] = \lim_{x\to5} 5f(x) - \lim_{x\to5} g(x)$

$= 5\lim_{x\to5} f(x) - \lim_{x\to5} g(x)$

$= 5(10) - (1) = 49$

The answer is D.

6. $$\lim_{x\to\infty} \frac{4x^3 + 3x^2 - x + 5}{6x^4 + x^3 - x^2 + 4x - 1} = \lim_{x\to\infty} \frac{\frac{4}{x} + \frac{3}{x^2} - \frac{1}{x^3} + \frac{5}{x^4}}{6 + \frac{1}{x} - \frac{1}{x^2} + \frac{4}{x^3} - \frac{1}{x^4}} = \frac{0 + 0 - 0 + 0}{6 + 0 - 0 + 0 - 0} = 0$$

The answer is B.

7. $\lim_{x\to\infty} \dfrac{3 + 4^x}{1 - 4^x} = \lim_{x\to\infty} \dfrac{\frac{3}{4^x} + 1}{\frac{1}{4^x} - 1} = \dfrac{1}{-1} = -1$

$\lim_{x\to-\infty} \dfrac{3 + 4^x}{1 - 4^x} = \dfrac{3 + 4^{-\infty}}{1 - 4^{-\infty}} = \dfrac{3}{1} = 3$

The horizontal asymptotes are $y = -1$ and $y = 3$, so the answer is C.

8. Evaluate each statement.

I. Because $\lim_{x\to2^-} g(x) = 1$ and $\lim_{x\to2^+} g(x) = 1$, $\lim_{x\to2} g(x) = 1$.

The statement is true.

II. $\lim_{x\to2} g(x) = 1 \neq g(2) = 3$

The statement is false.

III. g is continuous at $x = 3$.

The statement is true.

Because I and III are true, the answer is B.

9. Because $\lim_{x\to1^-} \dfrac{x-1}{\sqrt{x}-1} = 2$ and $\lim_{x\to1^+} \dfrac{x-1}{\sqrt{x}-1} = 2$, $\lim_{x\to1} \dfrac{x-1}{\sqrt{x}-1} = 2$.

The answer is C.

10. $\lim_{x\to\infty} \dfrac{x^3}{\sqrt{x^2+4}} = \lim_{x\to\infty} \dfrac{\frac{x^3}{x^3}}{\sqrt{\frac{x^2}{x^6} + \frac{4}{x^6}}}$

$= \lim_{x\to\infty} \dfrac{1}{\sqrt{\frac{1}{x^4} + \frac{4}{x^6}}}$

$= \dfrac{1}{0}$

$= \infty$

The answer is D.

11. (a) $\lim_{x\to 0} f(x) = \lim_{x\to 0} \dfrac{10}{1 + \frac{1}{4}e^{-x}}$

$= \dfrac{10}{1 + \frac{1}{4}}$

$= \dfrac{10}{\frac{5}{4}}$

$= 8$

1 pt: answer

(b) $\lim_{x\to 0} \left[f(x) + 4\right] = \lim_{x\to 0} f(x) + \lim_{x\to 0} 4$

$= 8 + 4$

$= 12$

1 pt: answer

(c) $\lim_{x\to \infty} f(x) = \lim_{x\to \infty} \dfrac{10}{1 + \frac{1}{4}e^{-x}}$

$= \lim_{x\to \infty} \dfrac{10}{1 + \frac{1}{4e^{x}}}$

$= \dfrac{10}{1 + \frac{1}{4e^{\infty}}}$

$= \dfrac{10}{1 + 0}$

$= 10$

$\lim_{x\to -\infty} f(x) = \lim_{x\to -\infty} \dfrac{10}{1 + \frac{1}{4}e^{-x}}$

$= \dfrac{10}{1 + \frac{1}{4}e^{\infty}}$

$= \dfrac{10}{\infty}$

$= 0$

So, the horizontal asymptotes are $y = 0$ and $y = 10$.

7 pts:
- 2 pts: examines both $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$
- 3 pts: computes each limit (answers with justification)
- 2 pts: writes equations of the horizontal asymptotes (must be equations)

Note: Evaluating limits would typically occur on only one or, at most, two parts of a free-response question (not as an entire question as practiced here).

12. (a) $f(x) = \dfrac{x^2 + 5x + 6}{2x^2 + 7x + 3} = \dfrac{(x + 2)(x + 3)}{(2x + 1)(x + 3)}$

$= \dfrac{x + 2}{2x + 1}, x \neq -3$

$f(x)$ has discontinuities at $x = -\dfrac{1}{2}$ and $x = -3$.

2 pts: answers

(b) $\lim\limits_{x\to-3} f(x) = \lim\limits_{x\to-3} \dfrac{x^2 + 5x + 6}{2x^2 + 7x + 3}$

$= \lim\limits_{x\to-3} \dfrac{(x + 2)\cancel{(x + 3)}}{(2x + 1)\cancel{(x + 3)}}$

$= \lim\limits_{x\to-3} \dfrac{x + 2}{2x + 1}$

$= \dfrac{-3 + 2}{2(-3) + 1}$

$= \dfrac{1}{5}$

2 pts: {1 pt: justification by factoring or L'Hôpital's Rule; 1 pt: answer}

(c) $f(x)$ has a vertical asymptote at $x = -\dfrac{1}{2}$.

1 pt: answer

Note: Including $x = -3$ in the answer (a removable discontinuity) would lose the answer point.

(d) $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \dfrac{x^2 + 5x + 6}{2x^2 + 7x + 3}$

$= \lim\limits_{x\to\infty} \dfrac{1 + \frac{5}{x} + \frac{6}{x^2}}{2 + \frac{7}{x} + \frac{3}{x^2}}$

$= \dfrac{1 + 0 + 0}{2 + 0 + 0}$

$= \dfrac{1}{2}$

$\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} \dfrac{x^2 + 5x + 6}{2x^2 + 7x + 3}$

$= \lim\limits_{x\to-\infty} \dfrac{1 + \frac{5}{x} + \frac{6}{x^2}}{2 + \frac{7}{x} + \frac{3}{x^2}}$

$= \dfrac{1 + 0 + 0}{2 + 0 + 0}$

$= \dfrac{1}{2}$

$f(x)$ has a horizontal asymptote at $y = \dfrac{1}{2}$.

4 pts:
- 1 pt: examines both $\lim\limits_{x\to\infty} f(x)$ and $\lim\limits_{x\to-\infty} f(x)$
- 2 pts: computes each limit (correct answer for each)
- 1 pt: writes equation of the horizontal asymptote (must be an equation)

Note: Examining only one of the limits would earn 0-1-1 at most.

13. (a) $\lim_{x\to 1}\left[f(x)+4\right] = \lim_{x\to 1} f(x) + \lim_{x\to 1} 4$
$= 2 + 4$
$= 6$

1 pt: answer

(b) $\lim_{x\to 3^-} \dfrac{5}{g(x)} = \dfrac{5}{1} = 5$

1 pt: answer

(c) $\lim_{x\to 2}\left[f(x)\cdot g(x)\right] = 2\cdot 0 = 0$

2 pts:
- 1 pt: justification (indicates the values of both $f(2)$ and $g(2)$; writing $2\cdot 0$ is sufficient)
- 1 pt: answer

(d) $\lim_{x\to 3}\dfrac{f(x)}{g(x)-1} = \lim_{x\to 3}\dfrac{(-2x+6)}{(x-2)-1}$
$= \lim_{x\to 3}\dfrac{-2x+6}{x-3}$
$= \lim_{x\to 3}\dfrac{-2\cancel{(x-3)}}{\cancel{(x-3)}}$
$= -2$

5 pts:
- 2 pts: finds linear equations for f and g on $[2, 3]$
- 2 pts: justification of limit by factoring or L'Hôpital's Rule
- 1 pt: answer

Note: The equations for f and g must both be correct to be eligible for the last three points (i.e., the limit must yield the indeterminant form $0/0$).

14. (a) $\lim_{x\to -1} f(x) = \lim_{x\to -1} e^{2x} = e^{2(-1)} = \dfrac{1}{e^2}$

1 pt: answer

(b) $f(0)$ is defined as $f(0) = e^{2(0)} = 1$.
$\lim_{x\to 0^-} f(x) = 1$ and $\lim_{x\to 0^+} f(x) = 1$,
so $\lim_{x\to 0} f(x) = 1$. Also,
$\lim_{x\to 0} f(x) = f(0) = 1$.
So, f is continuous at $x = 0$.

6 pts:
- 1 pt: finds $f(0)$
- 2 pts: finds each one-sided limit
- 1 pt: finds the limit (equates the values of the one-sided limits)
- 2 pts: indicates $f(0) = \lim_{x\to 0} f(x)$ to reach conclusion

(c) $\lim_{x\to -\infty} f(x) = \lim_{x\to -\infty} e^{2x}$
$= e^{2(-\infty)}$
$= \dfrac{1}{e^{\infty}}$
$= \dfrac{1}{\infty}$
$= 0$

2 pts: answer

15. (a) Because $T(x)$ is continuous on $[0, 10)$,

$\lim_{x \to 4} T(x) = T(4) = 172.$

2 pts: $\begin{cases} \text{1 pt: answer} \\ \text{1 pt: justification (appeals to the continuity of } T\text{)} \end{cases}$

(b) $$\frac{T(8) - T(3)}{8 - 3} = \frac{164 - 174}{8 - 3} = \frac{-10}{5} = -2$$

The average rate of change is −2°F per minute.

2 pts: $\begin{cases} \text{1 pt: justification (evidence of a difference quotient)} \\ \text{1 pt: answer with units} \end{cases}$

(c) $T(x)$ is continuous and when $x = 6$, $T(x) > 166.5°$ and when $x = 8$, $T(x) < 166.5°$.

So, the shortest interval is $(6, 8)$.

3 pts:
- 1 pt: shows $T(8) < 166.5 < T(6)$ (places 166.5 in this interval)
- 1 pt: answer (an open or closed interval would generally be accepted here)
- 1 pt: justification (appeals to the continuity of *T* or the Intermediate Value Theorem)

Note: Merely stating "because *T* is differentiable" or "because *T* is decreasing" would not earn the justification point.

(d) Because $T(x)$ is continuous, the average rate of change for $6 \le x \le 9$ is

$$\frac{T(9) - T(6)}{9 - 6} = \frac{162 - 168}{9 - 6} = \frac{-6}{3} = -2.$$

So, the tangent line at $x = 8$ has a slope of about -2.

2 pts:
- 1 pt: justification (evidence of a difference quotient on $[6, 9]$)
- 1 pt: answer

16. (a) Because $s(t)$ is a continuous function and $s(1) = 392$ and $s(2) = 377.3$, there exists a time t, $1 < t < 2$, where $s(t)$ is a value between 377.3 and 392.

2 pts:
- 1 pt: shows $s(2) < 382 < s(1)$ (places 382 in this interval)
- 1 pt: justification (appeals to the continuity of s or the Intermediate Value Theorem)

Note: Merely saying "because s is differentiable" or "because s is decreasing" would not earn the justification point.

(b)
$$s(t) = -4.9t^2 + 396.9$$
$$0 = -4.9t^2 + 396.9$$
$$4.9t^2 = 396.9$$
$$t^2 = 81$$
$$t = \pm 9$$

So, the object hits the ground after 9 seconds.

1 pt: answer with units

(c) $$\frac{s(9) - s(8)}{9 - 8} = \frac{0 - 83.3}{9 - 8} = \frac{-83.3}{1} = -83.3$$

The average rate of change is -83.3 meters per second.

This is a good measure of velocity because it is the object's average rate of change right before it hits the ground. It can be improved by finding the instantaneous rate of change using calculus.

4 pts:
- 1 pt: justification (evidence of a difference quotient on $[8, 9]$)
- 1 pt: answer with units
- 1 pt: explains why good estimate
- 1 pt: explains how to improve estimate (appeals to the use of an instantaneous rate of change or a smaller time interval)

(d)
$$\lim_{x \to 3} \frac{s(t) - s(3)}{t - 3}$$
$$= \lim_{x \to 3} \frac{(-4.9t^2 + 396.9) - [-4.9(3)^2 + 396.9]}{t - 3}$$
$$= \lim_{x \to 3} \frac{-4.9t^2 + 4.9(9)}{t - 3}$$
$$= \lim_{x \to 3} \frac{-4.9(t^2 - 9)}{t - 3}$$
$$= \lim_{x \to 3} \frac{-4.9\cancel{(t - 3)}(t + 3)}{\cancel{t - 3}}$$
$$= \lim_{x \to 3} -4.9(t + 3)$$
$$= -4.9(3 + 3)$$
$$= -29.4 \text{ m/sec}$$

2 pts:
- 1 pt: justification (by factoring or L'Hôpital's Rule)
- 1 pt: answer with units

17. (a) $f(x) = ax^2 + x - b$ $\qquad f(x) = ax + b$

$f(2) = a(2)^2 + (2) - b$ $\qquad f(2) = a(2) + b$

$= 4a + 2 - b$ $\qquad = 2a + b$

f is continuous at $x = 2$ when $4a + 2 - b = 2a + b$.

$f(x) = ax + b$ $\qquad f(x) = 2ax - 7$

$f(5) = a(5) + b$ $\qquad f(5) = 2a(5) - 7$

$= 5a + b$ $\qquad = 10a - 7$

f is continuous at $x = 5$ when $5a + b = 10a - 7$.

$4a + 2 - b = 2a + b \Rightarrow 2a - 2b = -2$

$5a + b = 10a - 7 \Rightarrow -5a + b = -7$

Multiply both sides of the second equation by 2.

$$\begin{array}{rcl} 2a - 2b &=& -2 \\ -10a + 2b &=& -14 \\ \hline -8a &=& -16 \\ a &=& 2 \end{array}$$

When $a = 2$, $b = 5(2) - 7 = 10 - 7 = 3$.

So, f is continuous when $a = 2$ and $b = 3$.

5 pts:
- 3 pts: equates $4a + 2 - b$ and $2a + b$ (equates the one-sided limits at $x = 2$) and equates $5a + b$ and $10a - 7$ (equates the one-sided limits at $x = 5$)
- 2 pts: answers (solves the system of equations to determine a and b)

(b) $\lim\limits_{x\to3} f(x) = \lim\limits_{x\to3} (2x + 3) = 2(3) + 3 = 9$

2 pts: answer from using values of a and b from part (a) and the correct piece of f

Note: Incorrect values of a and b may be imported from part (a) as long as a is nonzero (i.e., an answer consistent with values for part (a) can generally earn these two points).

(c)
$$\begin{aligned} \lim_{x\to1} g(x) &= \lim_{x\to1} \frac{f(x)}{x - 1} \\ &= \lim_{x\to1} \frac{2x^2 + x - 3}{x - 1} \\ &= \lim_{x\to1} \frac{(2x + 3)(x - 1)}{x - 1} \\ &= \lim_{x\to1} (2x + 3) \\ &= 2(1) + 3 \\ &= 5 \end{aligned}$$

2 pts:
- 1 pt: justification (by factoring or L'Hôpital's Rule)
- 1 pt: answer

Note: To be eligible for these points, the limit must yield the indeterminant form $\frac{0}{0}$ using the values of a and b from part (a) (i.e., incorrect values of a and b generally cannot be imported here).