AP® Exam Practice Questions for Chapter 2

1. $f(x) = 4e^x - x + 6$

$f'(x) = 4e^x - 1 + 0 = 4e^x - 1$

$f'(0) = 4e^0 - 1 = 4(1) - 1 = 3$

Tangent line: $y - 10 = 3(x - 0)$

$y = 3x + 10$

So, the answer is D.

2. Evaluate each graph.

A: The graph appears to be $f(x) = -2x$, so $f'(x) = -2$.

B: The graph appears to be $f(x) = x^3$, so $f'(x) = 3x^2$.

C: The graph appears to be $f(x) = x^2$, so $f'(x) = 2x$.

D: The graph appears to be $f(x) = x$, so $f'(x) = 1$.

Because $f'(x) = -2$ is negative for all values of x, the answer is A.

3.
$$y = \frac{6x^4 - 3x^5 + 5x^3}{x^3}$$
$$= 6x - 3x^2 + 5$$
$$\frac{dy}{dx} = 6 - 6x$$
$$\frac{d^2y}{dx} = -6$$

So, the answer is D.

4. The function $h(x) = |2x - 5|$ is continuous at $x = \frac{5}{2}$.

Because $\lim_{x \to (5/2)^-} \frac{|2x - 5|}{x - (5/2)} \neq \lim_{x \to (5/2)^+} \frac{|2x - 5|}{x - (5/2)}$, the function is not differentiable.

So, the answer is A.

5. $f(x) = \frac{\sin x}{x^2}$

$$f'(x) = \frac{x^2(\cos x) - \sin x(2x)}{(x^2)^2} = \frac{x \cos x - 2 \sin x}{x^3}$$

So, the answer is C.

6. $y = \sqrt[4]{8x + 3} = (8x + 3)^{1/4}$

$$y' = \frac{1}{4}(8x + 3)^{-3/4}(8) = \frac{2}{(8x + 3)^{3/4}}$$

So, the answer is A.

7. $y = 6 \cos 2x$

$y' = 6(-\sin 2x)(2) = -12 \sin 2x$

$y'' = -12 \cos 2x(2) = -24 \cos 2x$

$y''' = -24(-\sin 2x)(2) = 48 \sin 2x$

$y^{(4)} = 48 \cos 2x(2) = 96 \cos 2x$

$y^{(5)} = 96(-\sin 2x)(2) = -192 \sin 2x$

$y^{(6)} = -192 \cos 2x(2) = -384 \cos 2x$

So, the answer is B.

8. $\frac{s(3.2) - s(2.7)}{3.2 - 2.7} = \frac{10.6 - 7.8}{3.2 - 2.7} = \frac{2.8}{0.5} = 5.6$ m/sec

So, the answer is C.

9.
$$2y^3 - 3xy + x^2 = 4$$
$$6y^2\frac{dy}{dx} - 3x\frac{dy}{dx} - 3y + 2x = 0$$
$$(-3x + 6y^2)\frac{dy}{dx} = -2x + 3y$$
$$\frac{dy}{dx} = \frac{-(2x - 3y)}{(-3x + 6y^2)}$$
$$\frac{dy}{dx} = \frac{2x - 3y}{3x - 6y^2}$$

So, the answer is B.

10. $V = \pi r^2 h$

$$\frac{dV}{dt} = \pi\left[r^2\left(\frac{dh}{dt}\right) + 2r\left(\frac{dr}{dt}\right)h\right]$$
$$= \pi\left(r^2 \cdot \frac{dh}{dt} + 2rh\frac{dr}{dt}\right)$$

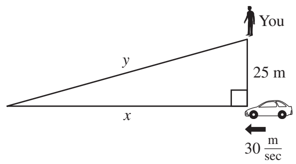
When $\frac{dr}{dt} = \frac{1}{3}$, $\frac{dh}{dt} = \frac{1}{2}$, $h = 9$, and $r = 4$,

$$\frac{dV}{dt} = \pi\left[(4)^2\left(\frac{1}{2}\right) + 2(4)(9)\left(\frac{1}{3}\right)\right]$$

$= 32\pi$ cubic centimeters per second.

So, the answer is D.

11.



$$x^2 + 25^2 = y^2$$

$$2x\frac{dx}{dt} + 0 = 2y\frac{dy}{dt}$$

$$\frac{dy}{dt} = \frac{x}{y}\frac{dx}{dt}$$

After 3 seconds, $x = 30(3) = 90$ meters and $y = \sqrt{90^2 + 25^2} = \sqrt{8725}$.

Because $\frac{dx}{dt} = 30$ meters per second, $\frac{dy}{dt} = \frac{90}{\sqrt{8725}}(30) \approx 28.906$ meters per second.

So, the answer is B.

12. $s(t) = -t^3 + 2t^2 + \frac{3}{2}$

$s'(t) = -3t^2 + 4t$

$$\text{Average velocity} = \frac{s(4) - s(0)}{4 - 0} = \frac{\left[-(4)^3 + 2(4)^2 + \frac{3}{2}\right] - \left[-(0)^3 + 2(0)^2 + \frac{3}{2}\right]}{4} = \frac{-32}{4} = -8$$

$$s'(t) = -8$$

$$-3t^2 + 4t = -8$$

$$0 = 3t^2 - 4t - 8$$

$$t = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-8)}}{2(3)}$$

$$t = \frac{4 \pm \sqrt{112}}{6}$$

$t \approx 2.431$ seconds $\quad (t \approx -1.097$ is not in the domain.)

So, the answer is D.

13. (a) $f(x) = 3e^{2x^2}$

$f'(x) = 3e^{2x^2}(4x) = 12xe^{2x^2}$

1 pt: answer

(b) Graph $f'(x) = 12xe^{2x^2}$ using a graphing utility. Use the *intersect* feature or the *root/zero* feature to find $f'(x) = 2$.

So, $x \approx 0.158$.

2 pts: $\begin{cases} \text{1 pt: sets } f'(x) = 2 \\ \text{1 pt: answer (using calculator — no work)} \end{cases}$

Notes: The answer must be rounded to at least three decimal places to receive the answer point. Avoid premature rounding in any intermediate steps.

To ensure the desired accuracy in the answer, solve $12xe^{2x^2} = 2$ numerically, perhaps using the *intersect* feature or the *root/zero* feature. Avoid using the *trace* feature because the desired level of accuracy may not be obtained without zooming in sufficiently.

An incorrect answer from part (a) may not be imported to part (b).

(c) Tangent line: $y - 0 = f'(c)\left(x - \frac{1}{2}\right)$

$$y = 12xe^{2x^2}\left(x - \frac{1}{2}\right)$$

Find x when $y = 3e^{2x^2}$.

$$3e^{2x^2} = 12xe^{2x^2}\left(x - \frac{1}{2}\right)$$

$$1 = 4x\left(x - \frac{1}{2}\right)$$

$$0 = 4x^2 - 2x - 1$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(4)(-1)}}{2(4)}$$

$$x = \frac{2 \pm \sqrt{20}}{8}$$

$$x = \frac{1 \pm \sqrt{5}}{4}$$

So, the tangent line intersects the x-axis at $x \approx 0.809$ and $x \approx -0.309$.

6 pts: $\begin{cases} \text{2 pts: represents an equation of the tangent line using } m = f'(x) = 12xe^{2x^2} \text{ (showing this in the next step is sufficient)} \\ \text{2 pts: represents the intersection of this tangent line with } f\left[\text{sets } 3e^{2x^2} \text{ equal to } 12xe^{2x^2}\left(x - \tfrac{1}{2}\right)\right] \\ \text{2 pts: answers (using calculator — no work)} \end{cases}$

Notes: The answers must be rounded to at least three decimal places to receive the answer points.

To ensure the desired accuracy in the answers, solve $3e^{2x^2} = 12xe^{2x^2}\left(x - \frac{1}{2}\right)$ or solve $3 = 12x\left(x - \frac{1}{2}\right)$ numerically, perhaps using the *intersect* feature or the *root/zero* feature. Avoid using the *trace* feature because the desired level of accuracy may not be obtained without zooming in sufficiently.

An incorrect answer from part (a) may be imported to part (c) to earn the first four points (but not the answer points).

14. (a) $\displaystyle\lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h}$

$$= \lim_{h\to 0}\frac{\sin x\cos h+\cos x\sin h-\sin x}{h}$$

$$= \lim_{h\to 0}\left[\frac{\sin x\cos h-\sin x}{h}+\cos x\left(\frac{\sin h}{h}\right)\right]$$

$$= \lim_{h\to 0}\left[\sin x\left(\frac{\cos h-1}{h}\right)+\cos x(1)\right]$$

$$= \lim_{h\to 0}\left(\sin x(0)+\cos x\right)$$

$$= \lim_{h\to 0}\cos x = \cos x$$

3 pts:
- 1 pt: expands $\sin(x+h)$
- 1 pt: manufactures the special limits $\displaystyle\lim_{h\to 0}\frac{\sin h}{h}$ and $\displaystyle\lim_{h\to 0}\frac{\cos h-1}{h}$
- 1 pt: answer with analysis

Note: A question requiring the use of such a trigonometric identity or these special limits is unlikely on the free–response section of the AP Exam.

(b) $$\lim_{h\to 0}\frac{\sqrt[3]{x+h}-\sqrt[3]{x}}{h}\left[\frac{\left(\sqrt[3]{x-h}\right)^2+\left(\sqrt[3]{x+h}\right)\left(\sqrt[3]{x}\right)+\left(\sqrt[3]{x}\right)^2}{\left(\sqrt[3]{x+h}\right)^2+\left(\sqrt[3]{x+h}\right)\left(\sqrt[3]{x}\right)+\left(\sqrt[3]{x}\right)^2}\right]$$

$$= \lim_{h\to 0}\frac{\left(\sqrt[3]{x+h}\right)^3-\left(\sqrt[3]{x}\right)^3}{h\left[\left(\sqrt[3]{x+h}\right)^2+\left(\sqrt[3]{x+h}\right)\left(\sqrt[3]{x}\right)+\left(\sqrt[3]{x}\right)^2\right]}$$

$$= \lim_{h\to 0}\frac{h}{h\left[\left(\sqrt[3]{x+h}\right)^2+\left(\sqrt[3]{x+h}\right)\left(\sqrt[3]{x}\right)+\left(\sqrt[3]{x}\right)^2\right]}$$

$$= \lim_{h\to 0}\frac{1}{\left(\sqrt[3]{x+h}\right)^2+\left(\sqrt[3]{x+h}\right)\left(\sqrt[3]{x}\right)+\left(\sqrt[3]{x}\right)^2}$$

$$= \frac{1}{x^{2/3}+x^{1/3}x^{1/3}+x^{2/3}}$$

$$= \frac{1}{3x^{2/3}}$$

2 pts:
- 1 pt: scales numerator and denominator by strategic factor
- 1 pt: answer with analysis

(c) $$\lim_{h\to 0}\frac{\sqrt{16+h}-4}{h}\left(\frac{\sqrt{16+h}+4}{\sqrt{16+h}+4}\right) = \lim_{h\to 0}\frac{(16+h)-16}{h\left(\sqrt{16+h}+4\right)}$$

$$= \lim_{h\to 0}\frac{h}{h\left(\sqrt{16+h}+4\right)}$$

$$= \lim_{h\to 0}\frac{1}{\sqrt{16+h}+4}$$

$$= \frac{1}{\sqrt{16}+4} = \frac{1}{8}$$

2 pts:
- 1 pt: scales numerator and denominator by strategic factor
- 1 pt: answer with analysis

(d) $$\lim_{h\to 0}\frac{\frac{1}{5+h}-\frac{1}{5}}{h} = \lim_{h\to 0}\frac{5-5-h}{5(5+h)}\cdot\frac{1}{h}$$

$$= \lim_{h\to 0}\frac{-1}{5(5+h)}$$

$$= -\frac{1}{5(5+0)}$$

$$= -\frac{1}{25}$$

2 pts:
- 1 pt: manufacturers a common denominator
- 1 pt: answer with analysis

Note: A question involving such limits as these four (i.e., evaluating the limit definition of the derivative) is unlikely on the free-response section of the AP Exam.

15. (a)

$$h(x) = \frac{f(x)}{g(x)}$$

$$h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

$$h'(2) = \frac{g(2)f'(2) - f(2)g'(2)}{[g(2)]^2}$$

$$= \frac{(5)(1) - (-3)(-2)}{5^2}$$

$$= -\frac{1}{25}$$

3 pts: $\begin{cases} \text{2 pts: uses the Quotient Rule to compute } h'(x) \\ \text{1 pt: answer} \end{cases}$

Note: Leaving the answer as $\frac{(5)(1) - (-3)(-2)}{5^2}$ is perfectly acceptable and, in fact, recommended. It is not necessary to simplify the answer on a free-response question. Students are encouraged to not simplify answers on the free-response questions to avoid the risk of making arithmetic mistakes.

(b)

$$j(x) = f(g(x))$$

$$j'(x) = f'(g(x))g'(x)$$

$$j'(2) = f'(g(2))g'(2)$$

$$= f'(5)(-2)$$

$$= (7)(-2)$$

$$= -14$$

3 pts: $\begin{cases} \text{2 pts: uses the Chain Rule to compute } j'(x) \\ \text{1 pt: answer} \end{cases}$

Note: Leaving the answer as $(7)(-2)$ is perfectly acceptable and, in fact, recommended. It is not necessary to simplify the answer on a free-response question.

(c)

$$k(x) = \sqrt{f(x)}$$

$$k'(x) = \frac{1}{2\sqrt{f(x)}} \cdot f'(x)$$

$$k'(5) = \frac{1}{2\sqrt{f(5)}} \cdot f'(5)$$

$$= \frac{1}{2\sqrt{4}} \cdot 7$$

$$= \frac{7}{4}$$

3 pts: $\begin{cases} \text{2 pts: uses the Chain Rule to compute } k'(x) \\ \text{1 pt: answer} \end{cases}$

Note: Leaving the answer as $\frac{1}{2\sqrt{4}} \cdot 7$ is perfectly acceptable and, in fact, recommended. It is not necessary to simplify the answer on a free-response question.

16. (a) (i) Because $v(t) > 0$ when $0 < t < 1$ and $4.4 < t < 5$, the particle is moving upward on the intervals $(0, 1)$ and $(4.4, 5)$.

(ii) Because $v(t) < 0$ when $2 < t < 4.4$, the particle is moving downward on the interval $(2, 4.4)$.

(iii) Because $v(t) = 0$ when $1 < t < 2$, the particle is at rest on the interval $(1, 2)$.

4 pts:
- 2 pts: (i) answers with explanation, where $v(t) > 0$
- 1 pt: (ii) answer with explanation, where $v(t) < 0$
- 1 pt: (iii) answer with explanation, where $v(t) = 0$

Notes: Though open intervals are desired, open or closed intervals would generally be accepted here.

For (i) and (ii): Because the zero of $v(t)$ on the interval $[4, 5]$ is being determined visually from the given graph, any t-value in the interval $[4.3, 4.5]$ would be acceptable for this zero. (Using exactly $t = 4.4$ in the answers is not required.)

(b) (i) When $t = 0.75$, the slope of the line of $v(t) = -2$. So, the acceleration of the particle is -2 feet per second squared.

(ii) When $t = 4.2$, the slope of the line of $v(t) = 5$. So, the acceleration of the particle is 5 feet per second squared.

5 pts:
- 2 pts: (i) answer with analysis (computes slope of this line segment)
- 2 pts: (ii) answer with analysis (computes slope of this line segment)
- 1 pt: units for both (i) and (ii)

Note: Leaving the answers in unsimplified forms, such as $\dfrac{2-0}{0-1}$ and $\dfrac{3-(-2)}{5-4}$, is perfectly acceptable and, in fact, recommended. It is not necessary to simplify the answers on a free-response question.

17. (a) $g(x) = f(x) \cdot \tan x + kx$

$g'(x) = f'(x) \cdot \tan x + \sec^2 x \cdot f(x) + k$

Because $\tan \dfrac{\pi}{2}$ and $\tan \dfrac{3\pi}{2}$ are undefined and $\sec^2 \dfrac{\pi}{2}$ and $\sec^2 \dfrac{3\pi}{2}$ are undefined, the derivative of g will fail to exist when $x = \dfrac{\pi}{2}$ and $x = \dfrac{3\pi}{2}$.

7 pts:
- 3 pts: uses the Product Rule to compute $g'(x)$
- 2 pts: answers
- 2 pts: justification (explaning $\tan x$ and/or $\sec x$ are undefined here)

(b) $g'(x) = f'(x) \cdot \tan x + \sec^2 x \cdot f(x) + k$

$$g'\left(\frac{\pi}{4}\right) = f'\left(\frac{\pi}{4}\right) \cdot \tan \frac{\pi}{4} + \left(\sec \frac{\pi}{4}\right)^2 \cdot f\left(\frac{\pi}{4}\right) + k$$

$$6 = (-2)(1) + \left(\sqrt{2}\right)^2(4) + k$$

$$6 = 6 + k$$

$$k = 0$$

2 pts:
- 1 pt: sets up correct equation
- 1 pt: answer (solves for k)

Note: Leaving the answer as $k = 6 - (-2)\tan \dfrac{\pi}{4} - \left(\sec \dfrac{\pi}{4}\right)^2(4)$ is perfectly acceptable and, in fact, recommended. It is not necessary to simplify the answer on a free-response question.